

---

## Presentation of Lucas Sequence with R

---

Senad Orhani\*

*\*Faculty of Education, University of Prishtina "Hasan Prishtina", Prishtina, Kosovo.*

*Corresponding Email: \*senad.orhani@uni-pr.edu*

**Received:** 08 February 2023

**Accepted:** 27 April 2023

**Published:** 02 June 2023

**Abstract:** *Lucas numbers form a sequence of numbers where each limit is the sum of the previous two numbers. It starts from the first limit with 2 and continues with the second limit with 1, like the first two limits of the sequence. Calculating future large numbers is complex and laborious in the Lucas sequence. Therefore, the purpose of this paper is to present the Lucas sequence with the help of technology, namely with the R program. The results show that the R program implemented with a simple algorithm gave us the results for the first 1001 numbers of the Lucas sequence.*

**Keywords:** *Math, Lucas Number, R Programming Language, and Sequence.*

### 1. INTRODUCTION

The Lucas sequence has many applications in many branches of mathematics such as set theory, calculus, applied mathematics, linear algebra, etc. [1]. During our study of the Fibonacci sequence, which can be observed in a number of amazing phenomena in nature [2], we also noticed similar applications in the Lucas sequence. By establishing important algebraic concepts, we will be able to create a ring that includes this group. Lucas sequences are subsets of a family of recursive sequences. A Lucas sequence is a sequence starting from 2 and 1, where the next terms can be obtained by adding the previous two consecutive terms [3]. The Lucas sequence forms the sequence with the numbers 2, 1, 3, 4, 7, 11, 18, 29, ... But, up to a point, the calculation becomes easy. But the question arises as to how the larger limits of this sequence can be calculated. Therefore, the purpose of this paper is that with the help of the R program it will be possible to present the Lucas sequence.

### 2. MATERIALS AND METHODS

#### Lucas numbers

Lucas numbers or Lucas sequence is a complete sequence named after the mathematician François Édouard Anatole Lucas (1842-1891), who studied both that sequence and numbers



closely related to the Fibonacci sequence. Lucas numbers and Fibonacci numbers form complementary examples of Lucas sequences [4].

**Definition 1:** The  $n^{\text{th}}$  Lucas number  $L_n$  is defined recursively by  $L_n = L_{n-1} + L_{n-2}$  for  $n \geq 2$  with initial terms  $L_0 = 2$  and  $L_1 = 1$  [5].

We can also define the Lucas sequence in this form:

$$L_n = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ L_{n-1} + L_{n-2} & \text{if } n > 1 \end{cases} \quad (1)$$

### R Programming Language

R data structures include vectors, sequences, lists, and data frames [6]. Vectors are ordered collections of values and can be arranged in arrays of one or more dimensions in a column-major order. That is, given an ordered collection of dimensions, the values along the first dimension are populated first, then the one-dimensional arrays along the second dimension are populated, and so on [7]. R supports array arithmetic and in this respect is like languages such as APL and MATLAB [6, 8]. The special case of a two-dimensional array is called a matrix. Lists serve as collections of objects that do not necessarily have the same data type. Data frames contain a list of vectors of the same length, plus a unique set of row names [6]. R has no scalar data type. Instead, a scalar is represented as a length-one vector [9].

R's capabilities are extended through user-created packages that provide statistical techniques, graphics facilities, import/export, reporting, and more. These packages, with their ease of installation and use, have been cited as driving widespread adoption of the language in data science. [10, 11, 12].

### 3. RESULTS

Each number from the Lucas sequence is defined according to formula 1, so their calculation can be done in this form:

$$n = 0, L_0 = 2$$

$$n = 1, L_1 = 1$$

$$n = 2, L_2 = L_{2-1} + L_{2-2} = L_1 + L_0 = 1 + 2 = 3$$

$$n = 3, L_3 = L_{3-1} + L_{3-2} = L_2 + L_1 = 3 + 1 = 4$$

$$n = 4, L_4 = L_{4-1} + L_{4-2} = L_3 + L_2 = 4 + 3 = 7$$

$$n = 5, L_5 = L_{5-1} + L_{5-2} = L_4 + L_3 = 7 + 4 = 11$$

$$n = 6, L_6 = L_{6-1} + L_{6-2} = L_5 + L_4 = 11 + 7 = 18$$

$$n = 7, L_7 = L_{7-1} + L_{7-2} = L_6 + L_5 = 18 + 11 = 29$$

$$n = 8, L_8 = L_{8-1} + L_{8-2} = L_7 + L_6 = 29 + 18 = 47$$

⋮

$$\begin{aligned} n = 999, L_{999} &= L_{999-1} + L_{999-2} = L_{998} + L_{997} \\ &= (3.712487e + 208) + (2.294443e + 208) = 6.006931e + 208 \end{aligned}$$

$$\begin{aligned} n = 1000, L_{1000} &= L_{1000-1} + L_{1000-2} = L_{999} + L_{998} \\ &= (6.006931e + 208) + (3.712487e + 208) = 9.719418e + 208 \end{aligned}$$



$$n = 1001, L_{1001} = L_{1001-1} + L_{1001-2} = L_{1000} + L_{999} \\ = (9.719418e + 208) + (6.006931e + 208) = 1.572635e + 209$$

The calculations involved above are somewhat complex and the risk of making a mistake is high. Also, calculating by hand is slow. So the technology has always been a source of some controversy. With the recent introduction in this paper to the representation of Lucas sequence numbers with the aid of the R program, perhaps it is time to review how and when to best use the technology to implement in the examples as it is made available for the Lucas sequence. Therefore, we present the code for running the Lucas sequence in R:

```
> lucas <- function(n){
+ phi <- (sqrt(5) + 1)/2
+ luc <- phi^n + (1-phi)^n
+ round(luc)
+ }
> lucas(0:1001)
```

The results of the Lucas sequence after running the code in R for the first 1001 numbers are presented in the following table:

Table 1. Lucas sequence for the first 1001 number

2.000000e+00	1.000000e+00	3.000000e+00	4.000000e+00	7.000000e+00
1.800000e+01	2.900000e+01	4.700000e+01	7.600000e+01	1.230000e+02
3.220000e+02	5.210000e+02	8.430000e+02	1.364000e+03	2.207000E+03
5.778000e+03	9.349000e+03	1.512700e+04	2.447600e+04	3.960300e+04
1.036820e+05	1.677610e+05	2.714430e+05	4.392040e+05	7.106470e+05
1.860498e+06	3.010349e+06	4.870847e+06	7.881196e+06	1.275204e+07
3.338528e+07	5.401852e+07	8.740380e+07	1.414223e+08	2.288261e+08
5.990746e+08	9.693230e+08	1.568398e+09	2.537721e+09	4.106118e+09
1.074996e+10	1.739380e+10	2.814375e+10	4.553755e+10	7.368130e+10
1.929002e+11	3.121190e+11	5.050192e+11	8.171382e+11	1.322157e+12
3.461453e+12	5.600748e+12	9.062201e+12	1.466295e+13	2.372515E+13
6.211325e+13	1.005014e+14	1.626146e+14	2.631160e+14	4.257306e+14
1.114577e+15	1.803424e+15	2.918001e+15	4.721424e+15	7.639425e+15
2.000027e+16	3.236112e+16	5.236140e+16	8.472252e+16	1.370839e+17
3.588904e+17	5.806968e+17	9.395871e+17	1.520284e+18	2.459871e+18
6.440026e+18	1.042018e+19	1.686021e+19	2.728039e+19	4.414060e+19
1.155616e+20	1.869826e+20	3.025441e+20	4.895267e+20	7.920708e+20
2.073668e+21	3.355266e+21	5.428934e+21	8.784200e+21	1.421313e+22
3.721047e+22	6.020780e+22	9.741827e+22	1.576261e+23	2.550444E+23
6.677148e+23	1.080385e+24	1.748100e+24	2.828485e+24	4.576585e+24
1.198166e+25	1.938673e+25	3.136838e+25	5.075511e+25	8.212349e+25
2.150021e+26	3.478807e+26	5.628828e+26	9.107634e+26	1.473646e+27
3.858056e+27	6.242466e+27	1.010052e+28	1.634299e+28	2.644351e+28
6.923000e+28	1.120165e+29	1.812465e+29	2.932630e+29	4.745095e+29



1.242282e+30	2.010055e+30	3.252337e+30	5.262391e+30	8.514728e+30
2.229185e+31	3.606896e+31	5.836081e+31	9.442978e+31	1.527906e+32
4.000109e+32	6.472313e+32	1.047242e+33	1.694474e+33	2.741716e+33
7.177905e+33	1.161409e+34	1.879200e+34	3.040609e+34	4.919809e+34
1.288023e+35	2.084065e+35	3.372088e+35	5.456152e+35	8.828240e+35
2.311263e+36	3.739702e+36	6.050966e+36	9.790668e+36	1.584163e+37
4.147394e+37	6.710624e+37	1.085802e+38	1.756864e+38	2.842666e+38
7.442196e+38	1.204173e+39	1.948392e+39	3.152565e+39	5.100957e+39
1.335448e+40	2.160800e+40	3.496248e+40	5.657048e+40	9.153296e+40
2.396364e+41	3.877398e+41	6.273762e+41	1.015116e+42	1.642492e+42
4.300101e+42	6.957709e+42	1.125781e+43	1.821552e+43	2.947333e+43
7.716217e+43	1.248510e+44	2.020132e+44	3.268642e+44	5.288774e+44
1.384619e+45	2.240361e+45	3.624980e+45	5.865340e+45	9.490320e+45
2.484598e+46	4.020164e+46	6.504762e+46	1.052493e+47	1.702969e+47
4.458430e+47	7.213892e+47	1.167232e+48	1.888621e+48	3.055854e+48
8.000328e+48	1.294480e+49	2.094513e+49	3.388994e+49	5.483507e+49
1.435601e+50	2.322851e+50	3.758451e+50	6.081302e+50	9.839754e+50
2.576081e+51	4.168186e+51	6.744267e+51	1.091245e+52	1.765672e+52
4.622590e+52	7.479507e+52	1.210210e+53	1.958160e+53	3.168370E+53
8.294901e+53	1.342143e+54	2.171633e+54	3.513776e+54	5.685409e+54
1.488460e+55	2.408378e+55	3.896838e+55	6.305216e+55	1.020205e+56
2.670932e+56	4.321659e+56	6.992591e+56	1.131425e+57	1.830684e+57
4.792793e+57	7.754903e+57	1.254770e+58	2.030260e+58	3.285029e+58
8.600319e+58	1.391561e+59	2.251593e+59	3.643154e+59	5.894746e+59
1.543265e+60	2.497055e+60	4.040319e+60	6.537374e+60	1.057769e+61
2.769276e+61	4.480783e+61	7.250059e+61	1.173084e+62	1.898090e+62
4.969264e+62	8.040438e+62	1.300970e+63	2.105014e+63	3.405984E+63
8.916983e+63	1.442798e+64	2.334496e+64	3.777294e+64	6.111791e+64
1.600088e+65	2.588996e+65	4.189084e+65	6.778080e+65	1.096716e+66
2.871241e+66	4.645765e+66	7.517006e+66	1.216277e+67	1.967978e+67
5.152232e+67	8.336487e+67	1.348872e+68	2.182521e+68	3.531393e+68
9.245306e+68	1.495922e+69	2.420452e+69	3.916374e+69	6.336827e+69
1.659003e+70	2.684323e+70	4.343326e+70	7.027649e+70	1.137097e+71
2.976960e+71	4.816822e+71	7.793782e+71	1.261060e+72	2.040439e+72
5.341938e+72	8.643437e+72	1.398537e+73	2.262881E+73	3.661418E+72
9.585718e+73	1.551002e+74	2.509574e+74	4.060575e+74	6.570149e+74
1.720087e+75	2.783160e+75	4.503247e+75	7.286407e+75	1.178965e+76
3.086571e+76	4.994177e+76	8.080749e+76	1.307493e+77	2.115568e+77
5.538628e+77	8.961688e+77	1.450032e+78	2.346200e+78	3.796232e+78
9.938664e+78	1.608110e+79	2.601976e+79	4.210086e+79	6.812062e+79
1.783421e+80	2.885636e+80	4.669057e+80	7.554692e+80	1.222375e+81
3.200219e+81	5.178063e+81	8.378282e+81	1.355635e+82	2.193463e+82
5.742560e+82	9.291657e+82	1.503422e+83	2.432587E+83	3.936009E+82



1.030461e+84	1.667320e+84	2.697781e+84	4.365101e+84	7.062882e+84
1.849086e+85	2.991885e+85	4.840971e+85	7.832856e+85	1.267383e+86
3.318051e+86	5.368719e+86	8.686770e+86	1.405549e+87	2.274226e+87
5.954001e+87	9.633776e+87	1.558778e+88	2.522155e+88	4.080933e+88
1.068402e+89	1.728711e+89	2.797113e+89	4.525824e+89	7.322937e+89
1.917170e+90	3.102046e+90	5.019216e+90	8.121262e+90	1.314048e+91
3.440222e+91	5.566396e+91	9.006617e+91	1.457301e+92	2.357963e+92
6.173227e+92	9.988492e+92	1.616172e+93	2.615021e+93	4.231193E+93
1.107741e+94	1.792362e+94	2.900103e+94	4.692465e+94	7.592568e+94
1.987760e+95	3.216263e+95	5.204023e+95	8.420287e+95	1.362431e+96
3.566891e+96	5.771350e+96	9.338241e+96	1.510959e+97	2.444783e+97
6.400526e+97	1.035627e+98	1.675679e+98	2.711306e+98	4.386985e+98
1.148528e+99	1.858357e+99	3.006885e+99	4.865241e+99	7.872126E+99
2.060949e+100	3.334686e+100	5.395635e+100	8.730322e+100	1.412596e+101
3.698224e+101	5.983851e+101	9.682075e+101	1.566593e+102	2.534800e+102
6.636193e+102	1.073759e+103	1.737378e+103	2.811136e+103	4.548514e+103
1.190817e+104	1.926782e+104	3.117598e+104	5.044380e+104	8.161978e+104
2.136834e+105	3.457469e+105	5.594303e+105	9.051772e+105	1.464607e+106
3.834392e+106	6.204177e+106	1.003857e+107	1.624275e+107	2.628131e+107
6.880538e+107	1.113294e+108	1.801348e+108	2.914642e+108	4.715991e+108
1.234662e+109	1.997726e+109	3.232388e+109	5.230114e+109	8.462502e+109
2.215512e+110	3.584773e+110	5.800285e+110	9.385058e+110	1.518534e+111
3.975574e+111	6.432615e+111	1.040819e+112	1.684080e+112	2.724899e+112
7.133879e+112	1.154286e+113	1.867674e+113	3.021960e+113	4.889633e+113
1.280123e+114	2.071282e+114	3.351405e+114	5.422686e+114	8.774091e+114
2.297087e+115	3.716765e+115	6.013851e+115	9.730616e+115	1.574447e+116
4.121955e+116	6.669463e+116	1.079142e+117	1.746088e+117	2.825230e+117
7.396548e+117	1.196787e+118	1.936442e+118	3.133228e+118	5.069670e+118
1.327257e+119	2.147547e+119	3.474803e+119	5.622350e+119	9.097153e+119
2.381666e+120	3.853616e+120	6.235281e+120	1.008890e+121	1.632418e+121
4.273726e+121	6.915033e+121	1.118876e+122	1.810379e+122	2.929255e+122
7.668889e+122	1.240852e+123	2.007741e+123	3.248594e+123	5.256335e+123
1.376126e+124	2.226619e+124	3.602746e+124	5.829365e+124	9.432110e+124
2.469359e+125	3.995506e+125	6.464865e+125	1.046037e+126	1.692524e+126
4.431084e+126	7.169645e+126	1.160073e+127	1.877037e+127	3.037110e+127
7.951258e+127	1.286541e+128	2.081666e+128	3.368207e+128	5.449873e+128
1.426795e+129	2.308603e+129	3.735399e+129	6.044002e+129	9.779401e+129
2.560280e+130	4.142621e+130	6.702901e+130	1.084552e+131	1.754842e+131
4.594237e+131	7.433631e+131	1.202787e+132	1.946150e+132	3.148937e+132
8.244023e+132	1.333911e+133	2.158313e+133	3.492224e+133	5.650537e+133
1.479330e+134	2.393606e+134	3.872936e+134	6.266542e+134	1.013948e+135
2.654550e+135	4.295152e+135	6.949702e+135	1.124485e+136	1.819456e+136
4.763396e+136	7.707337e+136	1.247073e+137	2.017807e+137	3.264880e+137



8.547568e+137	1.383026e+138	2.237782e+138	3.620808e+138	5.858590e+138
1.533799e+139	2.481739e+139	4.015538e+139	6.497276e+139	1.051281e+140
2.752290e+140	4.453299e+140	7.205590e+140	1.165889e+141	1.886448e+141
4.938785e+141	7.991121e+141	1.292991e+142	2.092103e+142	3.385093e+142
8.862289e+142	1.433949e+143	2.320178e+143	3.754126e+143	6.074304e+143
1.590273e+144	2.573116e+144	4.163390e+144	6.736506e+144	1.089990e+145
2.853630e+145	4.617270e+145	7.470900e+145	1.208817e+146	1.955907e+146
5.120631e+146	8.285355e+146	1.340599e+147	2.169134e+147	3.509732e+147
9.188599e+147	1.486747e+148	2.405606e+148	3.892353e+148	6.297959e+148
1.648827e+149	2.667858e+149	4.316686e+149	6.984544e+149	1.130123e+150
2.958700e+150	4.787278e+150	7.745978e+150	1.253326e+151	2.027923e+151
5.309172e+151	8.590421e+151	1.389959e+152	2.249001e+152	3.638961e+152
9.526923e+152	1.541489e+153	2.494181e+153	4.035669e+153	6.529850e+153
1.709537e+154	2.766089e+154	4.475626e+154	7.241715e+154	1.171734e+155
3.067640e+155	4.963545e+155	8.031185e+155	1.299473e+156	2.102592e+156
5.504656e+156	8.906721e+156	1.441138e+157	2.331810e+157	3.772947e+157
9.877704e+157	1.598246e+158	2.586017e+158	4.184263e+158	6.770279e+158
1.772482e+159	2.867936e+159	4.640419e+159	7.508355e+159	1.214877e+160
3.180590e+160	5.146303e+160	8.326893e+160	1.347320e+161	2.180009e+161
5.707337e+161	9.234666e+161	1.494200e+162	2.417667e+162	3.911867e+162
1.024140e+163	1.657094e+163	2.681234e+163	4.338327e+163	7.019561e+163
1.837745e+164	2.973534e+164	4.811279e+164	7.784813e+164	1.259609e+165
3.297699e+165	5.335790e+165	8.633489e+165	1.396928e+166	2.260277e+166
5.917482e+166	9.574686e+166	1.549217e+167	2.506685e+167	4.055902e+167
1.061849e+168	1.718108e+168	2.779957e+168	4.498065e+168	7.278021e+168
1.905411e+169	3.083019e+169	4.988430e+169	8.071449e+169	1.305988e+170
3.419121e+170	5.532254e+170	8.951374e+170	1.448363e+171	2.343500e+171
6.135363e+171	9.927226e+171	1.606259e+172	2.598982e+172	4.205241e+172
1.100946e+173	1.781368e+173	2.882315e+173	4.663683e+173	7.545998e+173
1.975568e+174	3.196536e+174	5.172104e+174	8.368640e+174	1.354074e+175
3.545013e+175	5.735951e+175	9.280964e+175	1.501692e+176	2.429788e+176
6.361267e+176	1.029275e+177	1.665401e+177	2.694676e+177	4.360078e+177
1.141483e+178	1.846958e+178	2.988442e+178	4.835400e+178	7.823842e+178
2.048308e+179	3.314233e+179	5.362541e+179	8.676773e+179	1.403931e+180
3.675540e+180	5.947149e+180	9.622689e+180	1.556984e+181	2.519253e+181
6.595489e+181	1.067173e+182	1.726722e+182	2.793894e+182	4.520616e+182
1.183513e+183	1.914963e+183	3.098476e+183	5.013440e+183	8.111916e+183
2.123727e+184	3.436263e+184	5.559990e+184	8.996252e+184	1.455624e+185
3.810874e+185	6.166123e+185	9.976997e+185	1.614312e+186	2.612012e+186
6.838335e+186	1.106466e+187	1.790299e+187	2.896765e+187	4.687065e+187
1.227089e+188	1.985472e+188	3.212562e+188	5.198034e+188	8.410596e+188
2.201923e+189	3.562786e+189	5.764708e+189	9.327494e+189	1.509220e+190
3.951190e+190	6.393160e+190	1.034435e+191	1.673751e+191	2.708186e+191





7.090123e+191	1.147206e+192	1.856218e+192	3.003424e+192	4.859642e+192
1.272271e+193	2.058578e+193	3.330848e+193	5.389426e+193	8.720274e+193
2.282997e+194	3.693968e+194	5.976965e+194	9.670933e+194	1.564790e+195
4.096673e+195	6.628556e+195	1.072523e+196	1.735378e+196	2.807901e+196
7.351181e+196	1.189446e+197	1.924564e+197	3.114010e+197	5.038574e+197
1.319116e+198	2.134374e+198	3.453490e+198	5.587865e+198	9.041355e+198
2.367057e+199	3.829979e+199	6.197037e+199	1.002702e+200	1.622405e+200
4.247512e+200	6.872619e+200	1.112013e+201	1.799275e+201	2.911288e+201
7.621851e+201	1.233241e+202	1.995427e+202	3.228668e+202	5.224095e+202
1.367686e+203	2.212962e+203	3.580648e+203	5.793610e+203	9.374258e+203
2.454213e+204	3.970999e+204	6.425212e+204	1.039621e+205	1.682142e+205
4.403906e+205	7.125669e+205	1.152957e+206	1.865524e+206	3.018482e+206
7.902488e+206	1.278649e+207	2.068898e+207	3.347548e+207	5.416446e+207
1.418044e+208	2.294443e+208	3.712487e+208	6.006931e+208	9.719418e+208

#### 4. CONCLUSION

In conclusion, in this paper the calculation strategy was not sufficient due to the size constraints and the complex nature of the mathematical operations involved in the Lucas sequence. The limits of the Lucas sequence begin with  $L_0 = 2$  and continue with  $L_1 = 1$ , following for  $n > 1$  given by the formula  $L_n = L_{n-1} + L_{n-2}$ . We tried to find the fastest and most accurate way to present the first 1001 limits of this sequence. Therefore, the novelty in this paper was the use of the R program to present the large limits of the Lucas sequence. The famous problem of determining all limits in the Lucas sequence was made possible by the algorithm implemented in this paper.

#### 5. REFERENCES

1. E. Özkant, M. Tastan dhe O. Gungor, «Catalan Transform of The  $k$  –Lucas Numbers,» Erzincan University Journal of Science and Technology, 2020.
2. S. Orhani, «Fibonacci Numbers as a Natural Phenomenon,» International Journal of Scientific Research and Innovative Studies, 1(1), 7-13, 2022.
3. U. Hasanah, S. Gemawati dhe S. Syamsudhuha, «Some Identities of Fibonacci-Like Based on Lucas Numbers,» International Journal of Mathematics Trends and Technology (IJMTT), 65(2), 37-40, 2019.
4. E. W. Weisstein, «Lucas Number,» 2022. [Në linjë]. Available: <https://mathworld.wolfram.com/LucasNumber.html>.
5. A. Dasdemir, «A study on the Jacobsthal and Jacobsthal-Lucas numbers,» DUFED, 3(1), 13-18, 2014.
6. P. Dalgaard, «Introductory Statistics with R,» New York, Berlin, Heidelberg: Springer-Verlag. 34, 10-18, 2002.
7. T. R. Davis, The Book of R, San Francisco, 2016.
8. H.-f. Chen, C. Wai-mee dhe Z. Da, «A Comparison Study on Execution Performance of MATLAB and APL,» 2022. [Në linjë].



8. R. Ihaka dhe R. Gentleman, «R: A Language for Data Analysis and Graphics,» Journal of Computational and Graphical Statistics, 5(3), 299-314, 1996.
9. J. M. Chambers, «S, R, and Data Science,» The R Journal. 12(1), 462–476, 2020.
10. Vance, «Data Analysts Captivated by R's Power,» 09 01 2009. [Në linjë]. Available: <https://www.nytimes.com/2009/01/07/technology/business-computing/07program.html>.
11. S. Tippmann, «Programming tools: Adventures with R,» Nature News. 517(7532), 109–110, 2014.