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Fatigue Analysis of an Inclined Crack Propagation Problem by the X-FEM Method

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Abstract: The extended finite element method (X-FEM) has been used to solve fracture mechanics, problems in materials with various behavior laws (for example, isotropic, orthotropic or piezoelectric materials... For each type of material, it is necessary to obtain "enrichment functions" which model the behavior of the fields of displacement and stresses in the vicinity of the front of crack. In this paper, fatigue crack propagation analysis was modeled, by the extended finite element method X-FEM to evaluate the total energy and strain energy at the angled crack length, and to have the development of the increment time concerning the different values of α which is equal to 15°, 30° and 45, this development has been studied numerically by solving the problem of finite elements by the computer code ABAQUS. Quadratic 4-node elements (CPS4R) were used.

***Keywords:** Analysis, Inclined, Crack Propagation, X-FEM Method, Increments.*

1. INTRODUCTION

The X-FEM extended finite element method, among the numerical modeling techniques used today. Thus is a technique based on the finite element method classical and the the partition of the unit method as well as on the nodes enrichment approach, Belytschko [1] is the first to develop the X-FEM. On the other hand, several researchers have used the X-FEM method to simulate fracture mechanics behavior in dynamic or stationary crack cases Hou et al [2] Moës and Belytschko [3], Wang et al [4], Chahine et al [5], Gupta et al [6], Budyn and Hoc [7]. Furthermore, this method has been used to summarize recent efforts by Bruce et al [8] in which to validate an X-FEM model regarding a cohesive mixed-mode PMMA fracture problem. El Fakkoussi et al [9] calculated the stress intensity factor KI, in mode I, by the FEM finite element method and the X-FEM extended finite element method in the linear elastic domain, of a longitudinal semi-elliptical crack of a tube. Bentahar et al [10] used the (X-FEM) method 2D the first mode I, to model crack propagation and energy evaluation

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(ALLSE) at the crack tip. On the other hand, Bentahar [11] used the finite element method (FEM) for anlysis the dissipation energy of a problem the crack propagation of a two-dimensions model.
Bakalakos et al [12] proposed domain decomposition solvers FETI-DP and P-FETI-DP to solve linear systems resulting from crack propagation analysis by XFEM method in 3D. Also, Guangwu et al [13] investigated the effect of interphase thickness on the propagation of a predefined matrix crack. On the other hand, A new method (SFEM) has been used by Bentahar et al [14], Bentahar and Benzaama [15] to characterize the stress intensity factors of an initial crack.

Crack Modeling With X-Fem
The modeling of a crack by X-FEM thus includes two types of enrichment: an enrichment for the front of the crack using functions characterizing the asymptotic behavior of the field of displacement close to the front of crack Belytschko and Black [1] and an enrichment for the interior of the crack using a function with value 1.0 above the crack and -1.0 below Moë et al [16]. The fact that a node is enriched or not and the type of enrichment depends on the relative position of its support compared to the crack.

The encircled nodes are enriched by discontinuity and the nodes surrounded by a square are enriched by the asymptotic modes in bottom of crack.

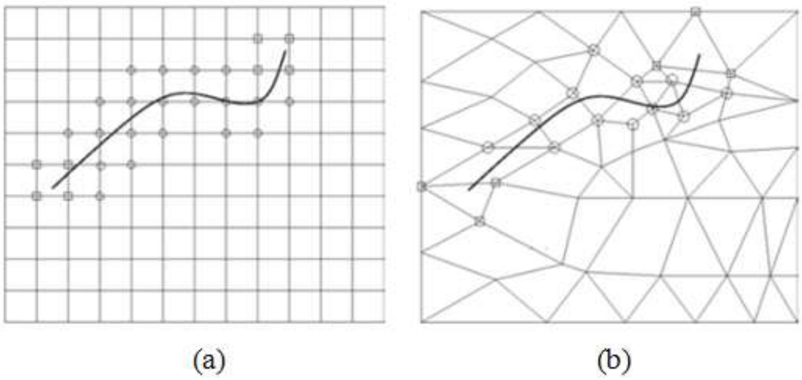

Fig. 1 Illustration of the X-FEM method: a) Crack placed on a uniform mesh and b) illustrates a non-uniform mesh [16]

L'approximation éléments finis enrichie pour modéliser la présence d'une fissure s'écrit :

$$\begin{aligned} \boldsymbol{u}^h(\boldsymbol{x}) &= \sum_{i\in I} \boldsymbol{u}_i \phi_i(\boldsymbol{x}) + \sum_{i\in L} \boldsymbol{a}_i \phi_i(\boldsymbol{x}) H(\boldsymbol{x}) \\ &+ \sum_{i\in K_1} \phi_i(\boldsymbol{x}) \Big(\sum_{l=1}^{4} \boldsymbol{b}^l_{i,1} F^l_1(\boldsymbol{x})\Big) + \sum_{i\in K_2} \phi_i(\boldsymbol{x}) \Big(\sum_{l=1}^{4} \boldsymbol{b}^l_{i,2} F^l_2(\boldsymbol{x})\Big) \end{aligned}$$

where:
$\boldsymbol{I}$ is the set of mesh nodes;
$\boldsymbol{u_i}$ is the classical (vectorial) degree of freedom at the node;

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Øi is the shape function (scalar) associated with the node;
L ∩ is the set of nodes enriched by the discontinuity and the coefficients are the degrees of freedom (corresponding verticals).

Elements Modeling
study has been proposed by Rahman and Siegfried [17] based on the X-FEM method of an Al 6061/ZrO2 composite material, to solve the effects of particles on the growth behavior of fatigue cracks.

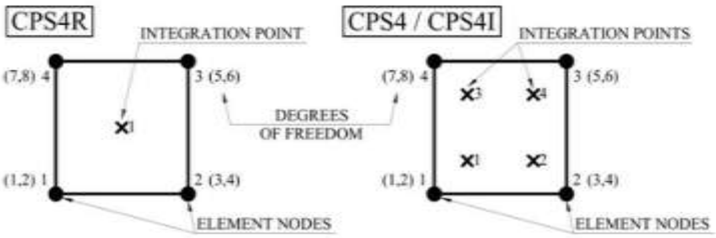

Fig. 2 Linear geometric order (first order) plane stress elements (author's study basing on Abaqus User's Manual) Kawecki and Podgórski [19]

On the other hand, the elements (CPE4R) with 4 bilinear nodes, reduced integration with piloting in hourglass, were used to model the effects of contact of a crack of a 2D model Bentahar et al [18].

Les différents modes de fissuration
En mécanique de la rupture, on distingue trois modes de rupture chacun d'eux :
Mode d'ouverture (mode I)
Mode cisaillement (mode II)
Mode anti-cisaillement plan (mode III).

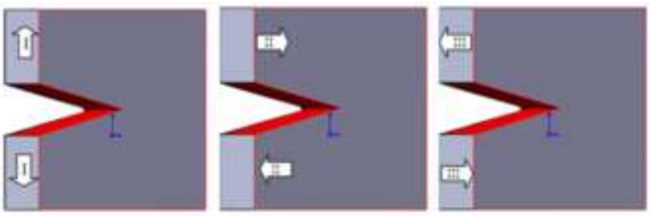

Fig. 3 Illustration of fracture cracking modes Bentahar [20]

Model And Matriel
The structure to be studied has the following dimensions: height R = 16 mm and width C = 8 mm, the length of the crack a = 1mm. The steel structure, the parameters of the elastic material are E = 72 Mpa and ν = 0.3. The structure is subjected to a uniform tensile stress σ = 100 Mpa, the embedding was applied on the lower surface of the structure. The crack is inclined by the angle α = 15°, 30° and 45°.

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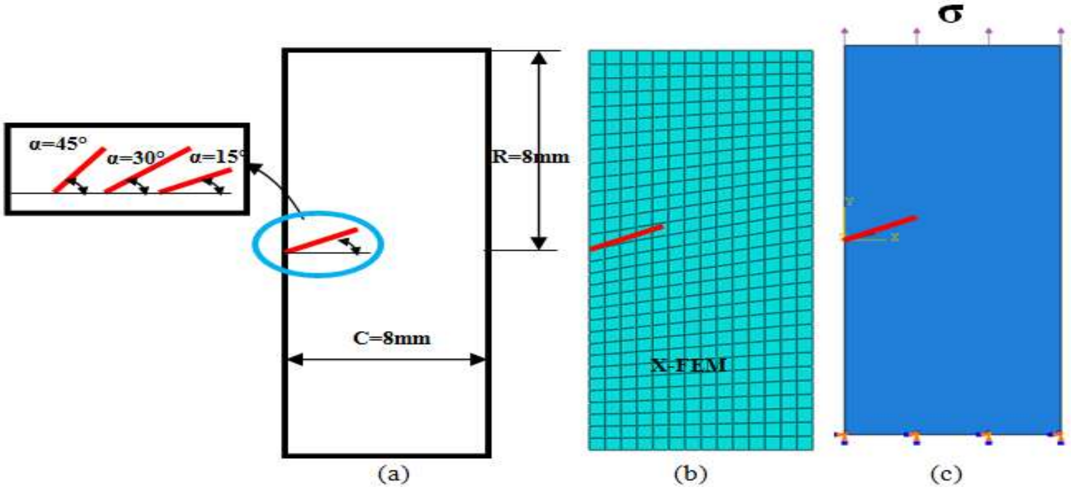

Fig. 4 X-FEM model: a) mesh and b) explains the different boundary conditions

Numerical modeling by XFEM method

The figure below presents a digital model modeled by the XFEM method concerning a crack inclined by the orientation angle α=15°, 30° and 45°. The different crack inclinations with the different increments have been presented by figures (6, 7, 8 and 9).

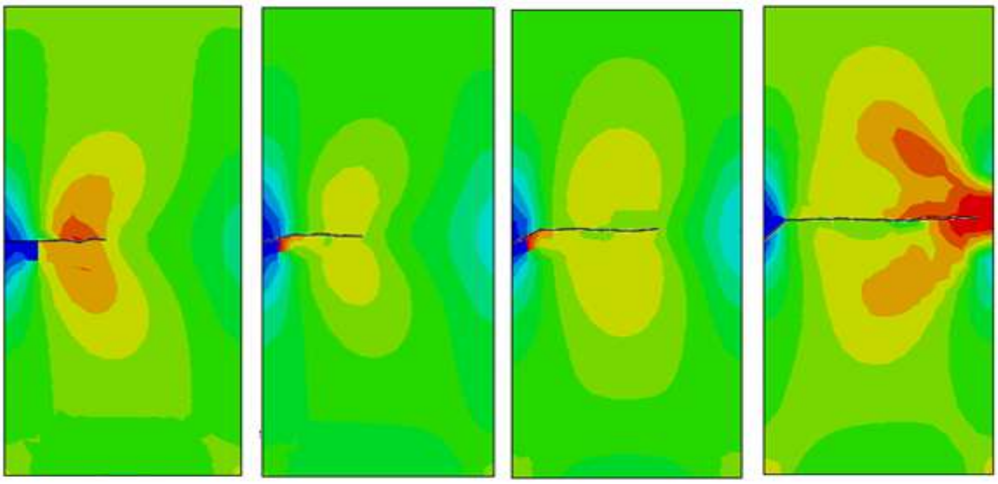

Fig. 5 Model of crack propagation by the X-FEM method
15° 30° 45°\

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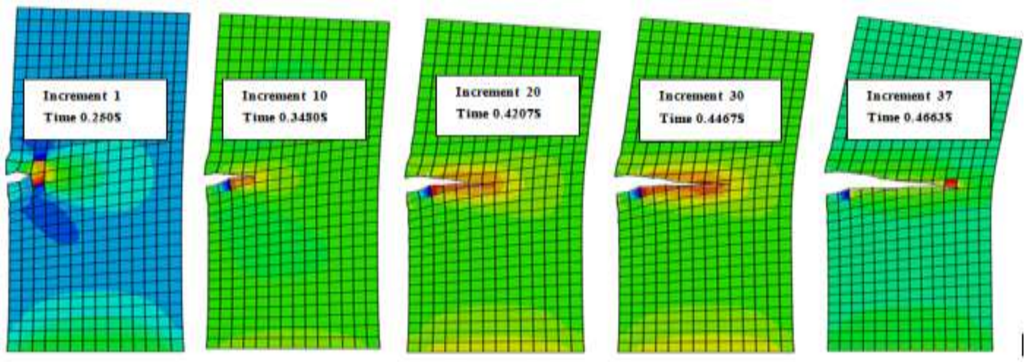

Fig. 6 Model X-FEM for a different inclination angle α a) 15° b) 30° c) 45°

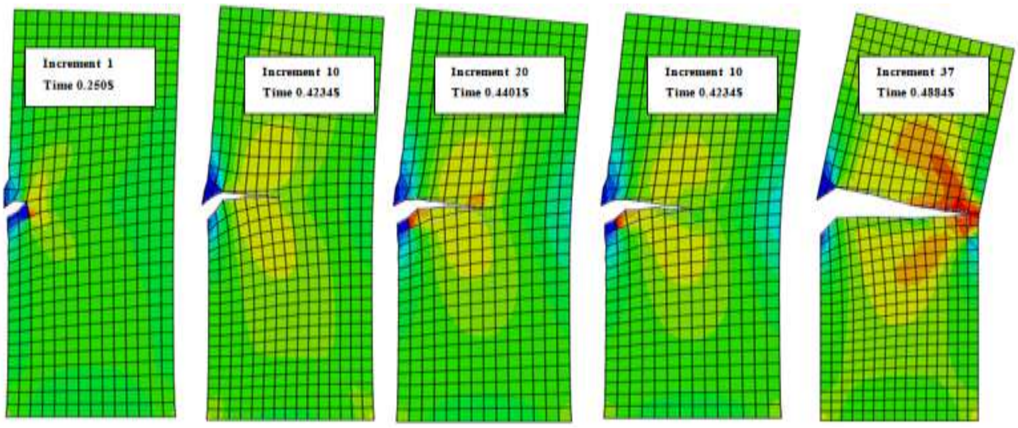

Fig. 7 numerical modeling by the X-FEM method of the model of crack inclined by the angle α= 15° concerning the increments 1, 10, 20, 30 and 37

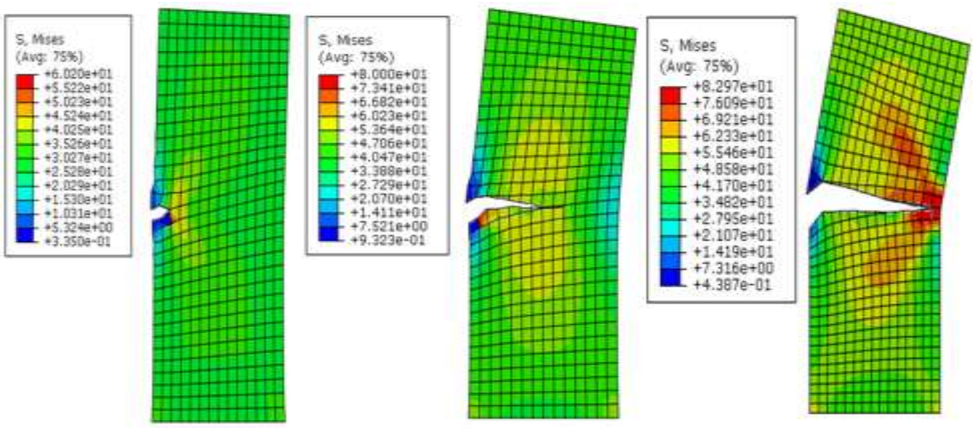

Fig. 8 numerical modeling by the X-FEM method of the model of crack inclined by the angle α= 30° concerning the increments 1, 10, 20, 30 and 37

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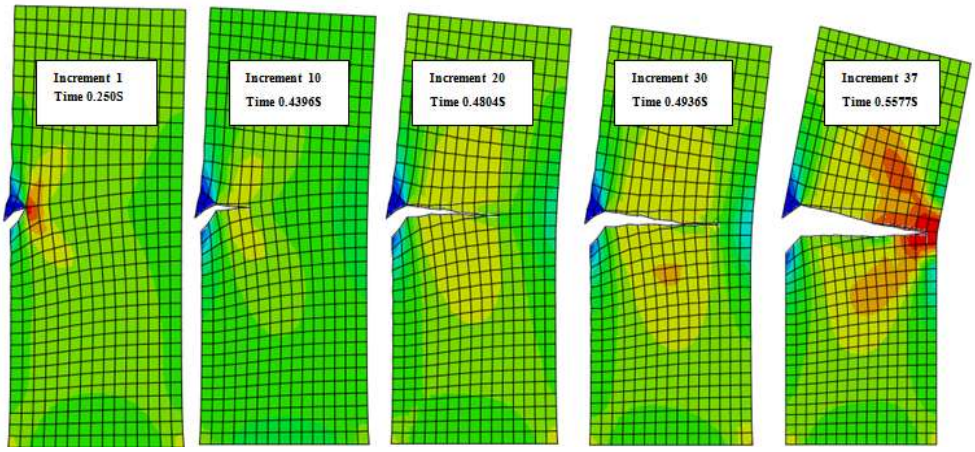

Fig. 9 numerical modeling by the X-FEM method of the model of crack inclined by the angle α= 45° concerning the increments 1, 10, 20, 30 and 37.

2. RESULTS AND DISCUSSION

The results and discussions of our work focused on the evolution of the total energy see figure 10 and the evolution of the strain energy see figure 11, of a model contains a crack inclined by the angle α =15°, 30° and 45°, using the X-FEM method.

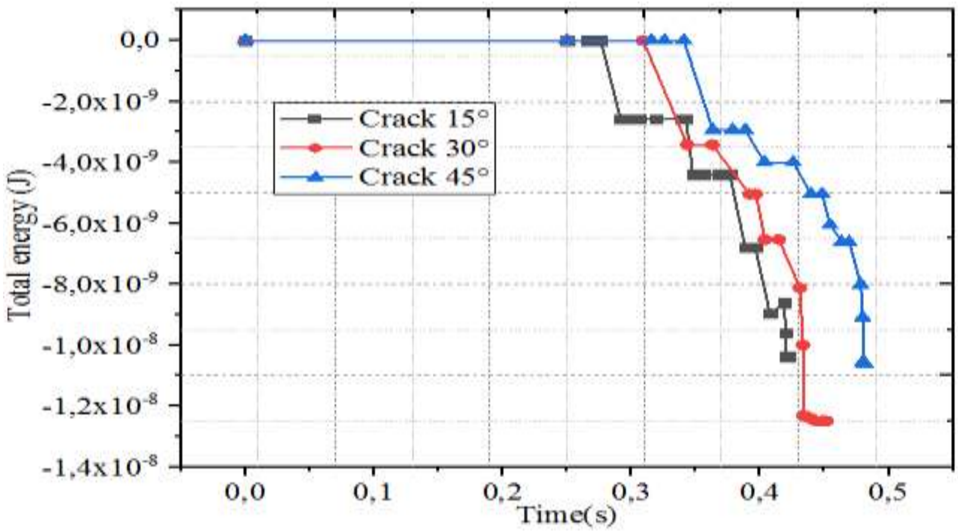

Fig. 10 Evolution of total energy function of time of inclined crack the d=15°, 30° and 45°

Figure 10 illustrates the evolution of the total energy when the cracked structure contains a crack inclined by three angles α=15°, 30° and 45°, respectively, the crack length a=1mm. It can be seen that the total energy remains constant at the zero level in the three cases of the crack inclination until time 0.25s after this time the total energy of the three models will start

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to decrease until the total energy value -1.4×10^{-8} J and time interval confined between 0.4s and 0.5s.

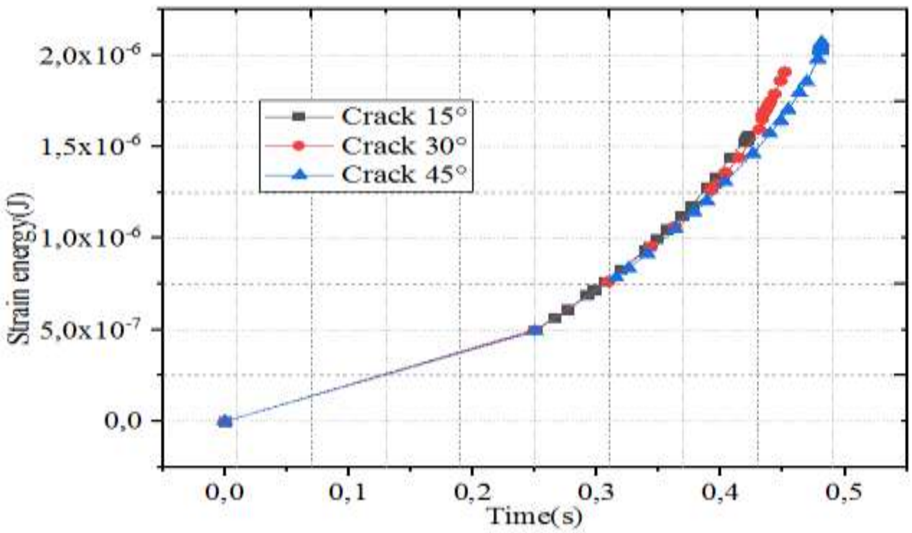

Fig. 11 Evolution of strain energy function of time of inclined crack the d=15°, 30° and 45°

The evolution of the strain energy has been illustrated in Figure 11. It can be seen that over a period of 0.5s, the strain energy tends to evolve and increase for the different study cases. Thus it is observed that the energy consumption almost the same during this period of time. Moreover, the interval of the variation of the deformation energy was varied between 0J and 2.0×10^{-6} J, and the results obtained and very proportional between the three models.

3. CONCLUSION

We conclude that: Three cases of initial crack propagation were studied, by the angle α = 15°, 30° and 45°.
The X-FEM extended finite element method was used to model the inclined crack of the three models.
It is observed that with the increase in time (t) the total energy decreases and the deformation energy increases concerning the different case studies.
The results obtained in our work justified, that the interval of the variation of the strain energy was varies between 0J and 2.0×10^{-6} J.
Furthermore, the variation of the total energy is varied between 0 and -1.4×10^{-8} J and the time interval confined between 0.4s and 0.5s.
The results obtained show that the strain energy varies slightly between the three cases and the results obtained between them are proportional. There is a proportionality between the results obtained from the two energies.

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