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Mathematical Modeling of the Dynamics of Lubrication

Waidi Adebayo Latifu*

**Faculty of Sciences, Mewar International University, Masaka, Nasarawa State, Nigeria*

*Corresponding Email: *latifwaid@yahoo.com*

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Abstract: A mathematical model is a model that is abstract and employs mathematical objects in order to explore and explain the real-life situation behaviour. The main thrust of the paper is to describe the dynamics of lubrication by assessing oil based on its viscosity .In this paper, a model is built using Reynolds equation together with the momentum and energy equations. Another model called Modulus equation is established by adopting experimental data .It is therefore shown that dependence of viscosity on temperature can be weakened when viscosity increases with growth of pressure.

Keywords: Engine Oil, Lubrication, Model, Viscosity.

1. INTRODUCTION

The development recorded in the globe in the centuries ago revealed the inevitability of applying mathematical methods and models in providing solutions to real life and life-long problems in and across different fields. Scholars thus tend to consider Mathematics responsive to solving problems of the real world situation; this is especially due to the increase in computational influence of digital computers and, thus begetting, complicated problems. The process involved in translating a real-life problem into a mathematical form can give an effective representation of and solution to certain problems. This process is known as Mathematical Modeling.
Engine oil comprises some important physical properties; the most important among is viscosity. This is because its viscosity has the basic importance for studying the components of lubrication and the efficiency of system.

The optimal viscosity of engine oil for a lubricating system is a compromise between the requirements of lubrication and the mechanical and volumetric efficiency [7]. The noble goal of using oil in engines and rotating machinery is lubrication. Lubrication is to form an oil layer or film between the surfaces so as to prevent the contact of metals. Oil also assists in cooling and

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processes of transferring the heat [5]. 10W-40 is one of several examples of engine oil, with the letter "W" meaning winter and the number showing that the oil contains the maximum viscosity at low temperature.

Background

Viscosity is the basic parameter of engine oil. Lubricating oil viscosity can be measured using many methods or techniques, and such techniques are grouped into static and dynamic tests. In static test, the viscosity is measured on the basis of oil conditions such as oil temperature. However in dynamic technique, the apparatus can apply load and torque on sample to simulate different operation conditions.

Vojtech Kumbar[12] employed a dynamic testing technique to measure the viscosity of different kinds of engine oils. He tested 10W-40 standard, semi-synthetic and synthetic types of oil and he then used the tested oil in four stroke motorcycle engines. The range of temperature during the experiment was from $5°C$ to $115°C$. The results of the temperature were used to create a mathematical formula for each oil type. The effect produced by mixing new and old oil was contained in [11]. The result indicated the engine oil for different mixing percentages. In his study, Udonne J.D. [10] did a comparative study of four methods of recycling the used lubrication oil to determine the change in viscosity. For oil recycling of waste engine oils [4] by acetic acid treatment, the process of treating the engine oil was developed for the application of automotive engines. In 2015, Yousef Alhouli et al [13] discussed the mathematical modeling of new and used engine oil using a classical method. In 2019, Hassan Lombera and J. Ignatio Tello[3] explored the theory of hydrodynamic lubrication as a phenomenon featured by a lubricant flowing in the narrow gap between two closely spaced surfaces in relative motion. In addition, Prashant Thapliyal and G.D. Thakre [9] studied the correlation of physiochemical, rheological and tribological parameters of engine oils. Lately this year 2022, a mathematical model [6] was developed for investigating the behaviours of nonlinear dynamic mixed friction which include contact force, deformation, hydrodynamic force and displacements of the novel coupled bearing lubricated with low-viscosity fluid.

2. METHODOLOGY

(a) Viscosity Dependence on Temperature

The lubricating oil viscosity is extremely sensitive to the temperature of the operation. Generally, with the increase in temperature, the oil viscosity falls rapidly. However, the following analysis of temperature influence on dynamic viscosity shows the weakness of this dependence.

There are several equations for viscosity-temperature; some are purely empirical while others are derived from theoretical models.

The Reynolds equation below is one of the most commonly used:

$$\mu = \mu_o e^{-\alpha T} \tag{2.1}$$

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where:
μ_o, α are constants
μ is the dynamic viscosity
T is the absolute temperature.
We consider the following momentum equation and energy equation respectively.

$$p\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=\frac{-\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2.2}$$

$$pc\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right)=\lambda\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

where:
u is the velocity of fluid
ρ is the density
p is the pressure
μ is the dynamic viscosity
c is the specific heat
T is the temperature
λ is the thermal conductivity
We operate wider under the following assumptions with a two-dimensional flow:
$u = u(y,t)$
$T = T(y,t)$
We assume symmetry i.e. $\frac{\partial u}{\partial x}=\frac{\partial T}{\partial x}=0$
$\frac{\partial p}{\partial x}=k=$ constant.

Under these assumptions and (2.1), the problem $((2.2)\,\&\,(2.3))$ becomes:

$$\rho\frac{\partial u}{\partial t}=-k+\frac{\partial}{\partial y}\left(\mu_o e^{-\alpha T}\frac{\partial u}{\partial y}\right) \tag{2.4}$$

$$\rho c\frac{\partial T}{\partial t}=\lambda\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.5}$$

Assume that the velocity is steady, so

$$k=\frac{\partial}{\partial y}\left(\mu_o e^{-\alpha T}\frac{\partial u}{\partial y}\right) \tag{2.6}$$

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$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\frac{\partial^2 T}{\partial y^2} + \mu_o \frac{e^{-\alpha T}}{\rho c}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.7)$$

Integrating (2.6),

$$ky + \text{constant} = \mu_o e^{-\alpha T}\frac{\partial u}{\partial y}$$

and so,

$$\frac{\partial u}{\partial y} = (ky + \text{constant})\frac{e^{\alpha T}}{\mu_o}$$

Assume that $\frac{\partial u}{\partial y} = 0$ when $y = 0$

$$0 = (0 + \text{constant})e^{\alpha T}$$

Therefore $\frac{\partial u}{\partial y} = ky\frac{e^{\alpha T}}{\mu_o}$ (2.8)

Substituting (2.8) into (2.5), we obtain

$$\rho c\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial y^2} + \mu_o e^{-\alpha T}\left(\frac{kye^{\alpha T}}{\mu_o}\right)^2$$

That is,

$$\rho c\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial y^2} + \frac{k^2y^2}{\mu_o}e^{\alpha T}$$

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} + \frac{k^2y^2}{\rho c\mu_o}e^{\alpha T}$$

Assume $\frac{\partial T}{\partial t} >> \frac{\partial^2 T}{\partial y^2}$, so $\frac{dT}{dt} = \frac{k^2y^2}{\rho c\mu_o}e^{\alpha T}$ (2.9)

Let $\frac{k^2y^2}{\rho c\mu_o} = 0$, (2.9) becomes $\frac{dT}{dt} = ae^{\alpha T}$ which gives the solution as $T = \frac{1}{\alpha}\log_e\left(\frac{1}{1-a\alpha t}\right)$.

As $1 - a\alpha t \to 0$ then.

And so,

$$\mu = \mu_o e^{-\infty} = 0 \qquad (2.10)$$

Hence, the viscosity of the engine decreases to zero. So the lubricating oil has to be changed. We had to make several assumptions to arrive at the result. We took $\frac{k^2y^2}{\mu_o\rho c}$ as a constant whereas y

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was a variable. Therefore, this assumption is only valid near the walls of the engine block that is in the areas $y = \pm 1$.

(b) **Pressure -Viscosity Coefficient**

The viscosity of lubricant oil increases with pressure growth.

The coefficient of the pressure-viscosity refers to the relationship between the load that is placed on the oil film (pressure) at the dynamic load zone and the thickness of oil film (viscosity) which is at that load, when all remaining factors (temperature, speed, material, geometry, load) are constant.

Barus equation is among the known equations employed to discuss the behaviour of the viscosity-pressure of the lubricant-oil [1]:

$$\mu = \mu_o e^{\alpha p}$$

where

μ is the dynamic viscosity of the pressure "p"

μ_o is the dynamic viscosity at the atmospheric pressure.

α is the pressure-viscosity coefficient

Taking α as

$$\alpha = \frac{1}{a_1 + a_2 T + (b_1 + b_2) p} \qquad (3.1)$$

where a_1, a_2, b_1, b_2 represent the oil behavior.

The Barus equation brings about a mathematical model known as "Modulus equation":

$$\mu = \mu_o e^{\left[\frac{p}{a_1 + a_2 T + (b_1 + b_2) p}\right]} \qquad (3.2)$$

At this stage, it is a common practice in the mining and construction industries to use engine oil SAE 10, SAE 20 or SAE 30 with the lowest API rating as a substitute for hydraulic oil ISO 32, ISO 46 or ISO 68 respectively for hydraulic systems of heavy equipment.

In view of this, parameters a_1, a_2, b_1, b_2 are calculated following the data given by the hydraulic oil procures [2].

$a_1 = 334 bar$

$a_2 = 3.2557\ bar/{}^oC$

$b_1 = 0.026266$

$b_2 = 0.000315$

For the pressure of $300 bar$ and temperature of 22.5^oC and 40^oC, formula (3.1) is used to evaluate the values of pressure-viscosity coefficient:

$$\alpha\left(p = 300 bar, T = 22.5^oC\right) = 0.00204844 bar^{-1}$$

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$\alpha(p = 300bar, T = 40^oC) = 0.0017959957bar^{-1}$

and values of dynamic viscosity are calculated using formula (3.2)

$\mu(p = 300bar, T = 22.5^oC) = 1.00205\mu_o$ and

$\mu(p = 300bar, T = 40^oC) = 1.00180\mu_o$.

The value of dynamic viscosity at pressure of $300bar$ and temperature $22.5(40^oC)$ is $1.00205(1.00180)$ times higher than the value of dynamic viscosity at atmospheric pressure and at similar temperature.

3. RESULTS

In this paper, two distinct methods were employed to show that it is possible to weaken the dependence of viscosity on temperature whereas viscosity increases with growth of pressure.

4. CONCLUSION

With our own approach, we succeeded in proving the correlation among viscosity, temperature and pressure. The relationship between the viscosity and pressure was established through an experimental data by previous mathematicians. We deemed it fit to propose that the later can also be proved by employing a more systematic method like that of temperature and viscosity.

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