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## Even Vertex in-Magic Total Labeling of Some 2-Regular Digraphs

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M.Sindhu<sup>1\*</sup>, S. Chandra Kumar<sup>2</sup>

<sup>1,2</sup>Research Scholar, Associate Professor, Department of Mathematics, Scott Christian College(Autonomous), Nagercoil, Tamilnadu, India.

Email: <sup>2</sup>kumar.chandra82@yahoo.com

Corresponding Email: <sup>1\*</sup>msindhu0387@gmail.com

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**Abstract:** Let  $D$  be a directed graph with  $p$  vertices and  $q$  arcs. A vertex in-magic total labeling (VIMTL) is a bijection  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, p + q\}$  with the property that for every  $v \in V(D)$ ,  $f(v) + \sum_{u \in I(v)} f((v, u)) = M$ , for some constant  $M$ . Such labeling is Even if  $f(V(D)) = \{2, 4, 6, \dots, 2p\}$ . In this paper, we explore the Even Vertex In-magic total labeling (EVIMTL) of some 2-regular directed graphs.

**Keywords:** Digraphs, Vertex In-Magic Labeling, Even Vertex In-Magic Total Labeling

### 1. INTRODUCTION

Graph Labeling is one of the most growing areas in graph theory. In graph theory, the labeling of graphs noticed to be a theoretical topic. It is used in countless applications like coding theory, X-Ray crystallography and astronomy etc. Design of graph labeling is helpful to network security, network addressing and social network in communication network. A magic total labeling of a graph is a motivating research area.

Throughout this paper,  $D = (V, A)$  is taken as a digraph with  $p$  vertices and  $q$  arcs. For a vertex  $v \in V(D)$ , the set  $I(v) = \{u | (u, v) \in A(D)\}$  is called the in-neighbourhood of  $v$ . The in-degree of  $v$  is defined by  $deg^-(v) = |I(v)|$ . A general reference for graph theoretic notions is [1].

A labeling of a graph  $G$  is a mapping from a set of vertices(edges) into a set of numbers, usually integers. Many kinds of labelings have been studied and an excellent survey of graph labelings can be found in [2].



In 1963, Sedláček<sup>[4]</sup> introduced the concept of magic labeling in graphs. A graph  $G$  is *magic* if the edges of  $G$  can be labelled by a set of numbers  $\{1, 2, \dots, q\}$  so that the sum of labels of all the edges incident with any vertex is the same.

In 2002, MacDougall et al.[3] introduced the notion of vertex magic total labeling (VMTL) in graphs. Let  $G(V, E)$  be a graph with  $|V(G)| = p$  and  $|E(G)| = q$ . A one-to-one map  $f$  from  $V(G) \cup E(G)$  onto the integers  $\{1, 2, \dots, p + q\}$  is a VTML if there is a constant  $M$  so that for every vertex  $x \in V(G)$ ,  $f(x) + \sum f(xy) = M$ , where the sum is taken over all vertices  $y$  adjacent to  $x$ .

In 2015, Alison M. Marr et al.[5] extended the study of VMTL in directed graphs, in the name 'vertex in-magic total labelings'. In the same year, Arumugam et al.[6] introduced the notion of E- super vertex in-magic total labeling (E-SVIMTL) in digraphs. A vertex in-magic total labeling of a digraph  $D$  is a bijection  $f$  from  $V(D) \cup A(D) \rightarrow \{1, 2, \dots, p + q\}$  with the property that for every vertex  $v \in V(D)$ ,  $f(v) + \sum_{u \in I(v)} f((v, u)) = M$ , for some constant  $M$ . Such a labeling is E-super vertex-in total (E-SVIMT) if  $f(A) = \{1, 2, 3, \dots, q\}$ . A digraph  $D$  which admits an E- SVIMTL is called an E-SVIMTL digraph.

CT. Nagaraj et al[7] in introduced the concept of an Even vertex magic total labeling. A vertex magic total labeling is even if  $f(V(G)) = \{2, 4, \dots, 2n\}$ . A graph is called an even vertex magic if the graph has an even vertex magic total labeling.

C.T Nagaraj et al[8] also studied Even vertex magic total labeling of some 2-regular graphs.

In this paper we define a new labeling called Even Vertex-In Magic Total Labeling (EVIMTL). An Even Vertex-In Magic Total Labeling (EVIMTL) is a bijection  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, p + q\}$  with the property  $f(V(D)) = \{2, 4, 6, \dots, 2p\}$  and for every  $v \in V(D)$ ,  $f(v) + \sum_{u \in I(v)} f((v, u)) = M$ , for some constant  $M$ . A digraph that admits an EVITML is called an Even vertex-in magic total (EVIMT). From the definition of EVITML, it is easy to observe that  $p \leq q$ .

## 2. MAIN RESULTS

In this section, we verify the existence of Even vertex-in magic total labeling for some 2-regular digraphs.

**Lemma 2.1.** If a digraph  $D(p, q)$  is an even vertex-in magic total (EVIMT), then the magic constant  $M$  is given by  $M = \frac{(p+q)(p+q+1)}{2p}$ .

**Proof:** Let  $f$  be an EVIMTL of  $D$ . Note that  $M = f(v) + \sum_{u \in I(v)} f((v, u))$  for all  $v \in V(D)$ . Summing over all  $v \in V(D)$ , we get



$$pM = \sum_{v \in V(D)} f(v) + \sum_{v \in V(D)} \sum_{u \in I(v)} f((v, u)) = \sum_{v \in V(D)} f(v) + \sum_{v \in V(D)} \sum_{u \in I(v)} f((v, u))$$

$$pM = [1 + 3 + \dots + 2p - 1] + [2 + 4 + \dots + 2p] + [1 + 2 + \dots + (p + q)] - [1 + 2 + \dots + 2p]$$

$$pM = [1 + 2 + \dots + (p + q)] = \frac{(p+q)(p+q+1)}{2} \text{ and}$$

$$\text{Hence } M = \frac{(p+q)(p+q+1)}{2p}.$$

**Theorem 2.1** The digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t}}, t \geq 1$  admits EVIMTL with magic constant  $8t + 7$ .

**Proof:**

Let the  $V(D) = \{a_i: 1 \leq i \leq 3\} \cup \{b_i: 1 \leq i \leq 4t\}$  and  $A(D) = \{(a_i, a_{i \oplus_3 1}): 1 \leq i \leq 3\} \cup \{(b_i, b_{i \oplus_{4t} 1}): 1 \leq i \leq 4t\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 8t + 6\}$  as follows

$$f(u) = \begin{cases} 8t + 2i & \text{if } u = a_i \text{ for } 1 \leq i \leq 3 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 4t \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 7 - 2i & \text{if } e = (a_i, a_{i \oplus_3 1}) \text{ for } 1 \leq i \leq 3 \\ 8t + 7 - 2i & \text{if } e = (b_i, b_{i \oplus_{4t} 1}) \text{ for } 1 \leq i \leq 4t \end{cases}$$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$

Then

$$f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_3 1})) = [8t + 2i] + [7 - 2i] = 8t + 7.$$

**Case 2:** Suppose  $v = b_i$

Then

$$f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{4t} 1})) = [2i] + [8t + 7 - 2i] = 8t + 7.$$

The digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t}}, t \geq 1$  admits EVIMTL with the magic constant  $8t + 7$ .

**Example 2.2** Consider the digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t}}$  taking  $t = 4$ .

Here  $V(D) = \{a_i: 1 \leq i \leq 3\} \cup \{b_i: 1 \leq i \leq 16\}$  and  $A(D) = \{(a_i, a_{i \oplus_3 1}): 1 \leq i \leq 3\} \cup \{(b_i, b_{i \oplus_{16} 1}): 1 \leq i \leq 16\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 38\}$  as follows

$$f(u) = \begin{cases} 32 + 2i & \text{if } u = a_i \text{ for } 1 \leq i \leq 3 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 16 \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 7 - 2i & \text{if } e = (a_i, a_{i \oplus_3 1}): 1 \leq i \leq 3 \\ 39 - 2i & \text{if } e = (b_i, b_{i \oplus_{16} 1}) \text{ for } 1 \leq i \leq 16 \end{cases}$$

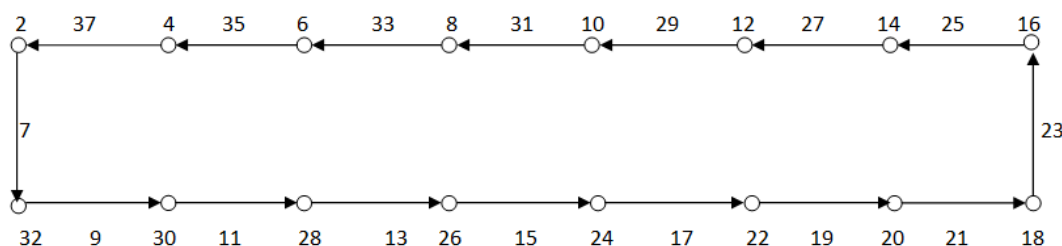
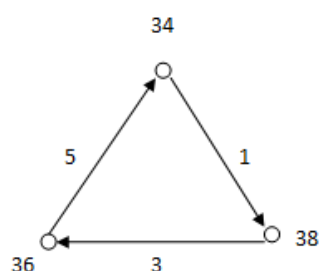


Figure 1:  $\vec{C}_3 \cup \vec{C}_{16}$   $k = 39$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$  for  $i = 1, 2, 3$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_3 1})) = [32 + 2i] + [7 - 2i] = 39.$$

**Case 2:** Suppose  $v = b_i$  for  $1 \leq i \leq 16$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{16} 1})) = [2i] + [39 - 2i] = 39.$$

The graph  $C_3 \cup C_{4t}$ ,  $t \geq 1$  is EVIMT with the magic constant 39.

**Theorem 2.3** The digraph  $D = \vec{C}_3 \cup \vec{C}_{4t+2}$ ,  $t \geq 1$  admits EVIMTL with magic constant  $8t + 11$ .



**Proof:**

Let the  $V(D) = \{a_i: 1 \leq i \leq 3\} \cup \{b_i: 1 \leq i \leq 4t + 2\}$  and  $A(D) = \{(a_i, a_{i \oplus_3 1}): 1 \leq i \leq 3\} \cup \{(b_i, b_{i \oplus_{4t+2} 1}): 1 \leq i \leq 4t + 2\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 8t + 10\}$  as follows

$$f(u) = \begin{cases} 8t + 2i + 4 & \text{if } u = a_i \text{ for } 1 \leq i \leq 3 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 4t + 2 \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 7 - 2i & \text{if } e = (a_i, a_{i \oplus_3 1}): 1 \leq i \leq 3 \\ 8t + 11 - 2i & \text{if } e = (b_i, b_{i \oplus_{4t+2} 1}) \text{ for } 1 \leq i \leq 4t + 2 \end{cases}$$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$  for  $1 \leq i \leq 3$

Then  $f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_3 1})) = [8t + 2i + 4] + [7 - 2i] = 8t + 11$ .

**Case 2:** Suppose  $v = b_i$  for  $1 \leq i \leq 4t + 2$

Then  $f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{4t+2} 1})) = [2i] + [8t + 11 - 2i] = 8t + 11$ .

The graph  $C_3 \cup C_{4t+2}, t \geq 1$  admits EVIMTL with the magic constant  $8t + 11$ .

**Example 2.4** Consider the digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t+2}}$  taking  $t = 4$ .

Here  $V(D) = \{a_i: 1 \leq i \leq 3\} \cup \{b_i: 1 \leq i \leq 18\}$  and  $A(D) = \{(a_i, a_{i \oplus_3 1}): 1 \leq i \leq 3\} \cup \{(b_i, b_{i \oplus_{18} 1}): 1 \leq i \leq 18\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 42\}$  as follows

$$f(u) = \begin{cases} 36 + 2i & \text{if } u = a_i \text{ for } 1 \leq i \leq 3 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 18 \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 7 - 2i & \text{if } e = (a_i, a_{i \oplus_3 1}) \text{ for } 1 \leq i \leq 3 \\ 43 - 2i & \text{if } e = (b_i, b_{i \oplus_{18} 1}) \text{ for } 1 \leq i \leq 18 \end{cases}$$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$  for  $1 \leq i \leq 3$

Then

$$f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_3 1})) = [36 + 2i] + [7 - 2i] = 43.$$

**Case 2:** Suppose  $v = b_i$  for  $1 \leq i \leq 4t + 2$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{18} 1})) = [2i] + [43 - 2i] = 43.$$

The graph  $C_3 \cup C_{4t+2}$ ,  $t \geq 1$  is EVIMT with the magic constant 43.

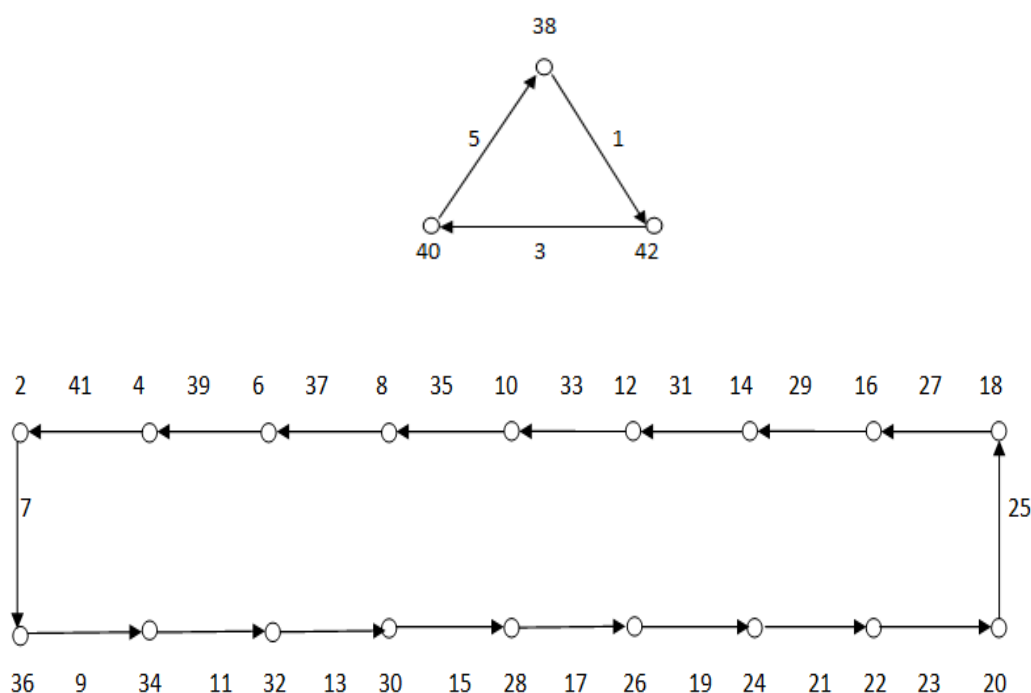


Figure 2:  $\vec{C}_3 \cup \vec{C}_{18}$   $k = 43$

**Theorem 2.5** The digraph  $D = \vec{C}_4 \cup \vec{C}_{4t+3}$ ,  $t \geq 1$  admits EVIMTL with magic constant  $8t + 15$ .

**Proof:**

Let the  $V(D) = \{a_i: 1 \leq i \leq 4\} \cup \{b_i: 1 \leq i \leq 4t + 3\}$  and  $A(D) = \{(a_i, a_{i \oplus_4 1}): 1 \leq i \leq 4\} \cup \{(b_i, b_{i \oplus_{4t+3} 1}): 1 \leq i \leq 4t + 3\}$  be the vertex set and arc set of  $D$  respectively.



Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 8t + 14\}$  as follows

$$f(u) = \begin{cases} 8t + 2i + 6 & \text{if } u = a_i \text{ for } 1 \leq i \leq 4 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 4t + 3 \end{cases}$$

and

$$f(e) = \begin{cases} 9 - 2i & \text{if } e = (a_i, a_{i \oplus_4 1}): 1 \leq i \leq 4 \\ 8t + 15 - 2i & \text{if } e = (b_i, b_{i \oplus_{4t+3} 1}) \text{ for } 1 \leq i \leq 4t + 3. \end{cases}$$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i, 1 \leq i \leq 4$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_4 1})) = [8t + 2i + 6] + [9 - 2i] = 8t + 15.$$

**Case 2:** Suppose  $v = b_i, 1 \leq i \leq 4t + 3$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{4t+3} 1})) = [2i] + [8t + 15 - 2i] = 8t + 15.$$

The digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t}}$ ,  $t \geq 1$  admits EVIMTL with the magic constant  $8t + 15$ .

**Example 2.6** Consider the digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t+3}}$  taking  $t = 3$

Here  $V(D) = \{a_i: 1 \leq i \leq 4\} \cup \{b_i: 1 \leq i \leq 15\}$  and  $A(D) = \{(a_i, a_{i \oplus_4 1}): 1 \leq i \leq 4\} \cup \{(b_i, b_{i \oplus_{15} 1}): 1 \leq i \leq 15\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 38\}$  as follows

$$f(u) = \begin{cases} 30 + 2i & \text{if } u = a_i \text{ for } 1 \leq i \leq 4 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 15 \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 9 - 2i & \text{if } e = (a_i, a_{i \oplus_4 1}): 1 \leq i \leq 4 \\ 39 - 2i & \text{if } e = (b_i, b_{i \oplus_{15} 1}) \text{ for } 1 \leq i \leq 15 \end{cases}$$

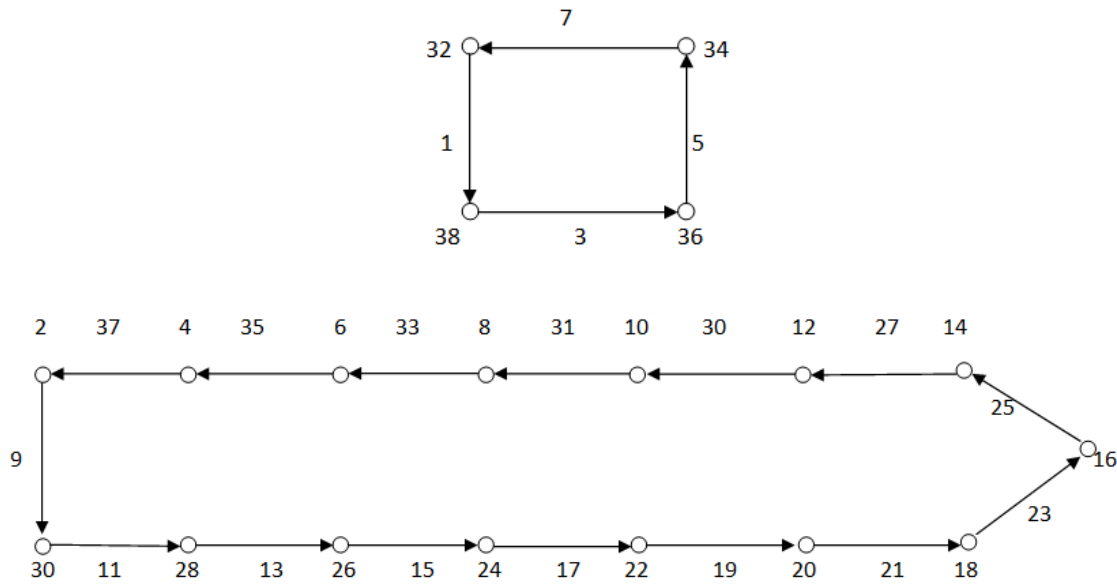


Figure 3:  $\vec{C}_4 \cup \vec{C}_{15}$   $k = 39$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$  for  $1 \leq i \leq 4$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_4 1})) = [30 + 2i] + [9 - 2i] = 39.$$

**Case 2:** Suppose  $v = b_i$  for  $1 \leq i \leq 15$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{15} 1})) = [2i] + [39 - 2i] = 39.$$

The graph  $C_3 \cup C_{4t+3}, t \geq 1$  is EVIMT with the magic constant 39.

**Theorem 2.7** The digraph  $D = \vec{C}_3 \cup \vec{C}_{4t+1}, t \geq 1$  admits EVIMTL with magic constant  $8t + 11$ .

**Proof:**

Let the  $V(D) = \{a_i: 1 \leq i \leq 4\} \cup \{b_i: 1 \leq i \leq 4t + 1\}$  and  $A(D) = \{(a_i, a_{i \oplus_4 1}): 1 \leq i \leq 4\} \cup \{(b_i, b_{i \oplus_{4t+1} 1}): 1 \leq i \leq 4t + 1\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 8t + 10\}$  as follows





$$f(u) = \begin{cases} 8t + 2i + 2 & \text{if } u = a_i \text{ for } 1 \leq i \leq 4 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 4t + 1 \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 9 - 2i & \text{if } e = (a_i, a_{i \oplus_4 1}): 1 \leq i \leq 4 \\ 8t + 11 - 2i & \text{if } e = (b_i, b_{i \oplus_{4t+1} 1}) \text{ for } 1 \leq i \leq 4t + 1 \end{cases}$$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$  for  $1 \leq i \leq 4$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_3 1})) = [8t + 2i + 4] + [9 - 2i] = 8t + 11.$$

**Case 2:** Suppose  $v = b_i$  for  $1 \leq i \leq 4t + 1$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{4t+1} 1})) = [2i] + [8t + 11 - 2i] = 8t + 11.$$

The graph  $C_3 \cup C_{4t+2}, t \geq 1$  admits EVIMTL with the magic constant  $8t + 11$ .

**Example 2.4** Consider the digraph  $D = \overrightarrow{C_3} \cup \overrightarrow{C_{4t+1}}$  taking  $t = 3$ .

Here  $V(D) = \{a_i: 1 \leq i \leq 4\} \cup \{b_i: 1 \leq i \leq 13\}$  and  $A(D) = \{(a_i, a_{i \oplus_3 1}): 1 \leq i \leq 4\} \cup \{(b_i, b_{i \oplus_{13} 1}): 1 \leq i \leq 13\}$  be the vertex set and arc set of  $D$  respectively.

Define  $f: V(D) \cup A(D) \rightarrow \{1, 2, \dots, 34\}$  as follows

$$f(u) = \begin{cases} 26 + 2i & \text{if } u = a_i \text{ for } 1 \leq i \leq 4 \\ 2i & \text{if } u = b_i \text{ for } 1 \leq i \leq 13 \end{cases} \quad \text{and}$$

$$f(e) = \begin{cases} 9 - 2i & \text{if } e = (a_i, a_{i \oplus_4 1}) \text{ for } 1 \leq i \leq 4 \\ 35 - 2i & \text{if } e = (b_i, b_{i \oplus_{13} 1}) \text{ for } 1 \leq i \leq 13 \end{cases}$$

Now we prove  $f$  is an EVIMTL

**Case 1:** Suppose  $v = a_i$  for  $1 \leq i \leq 4$

$$\text{Then } f(v) + \sum_{u \in I(v)} f((v, u)) = f(a_i) + f((a_i, a_{i \oplus_4 1})) = [26 + 2i] + [9 - 2i] = 35.$$

**Case 2:** Suppose  $v = b_i$  for  $1 \leq i \leq 13$

Then  $f(v) + \sum_{u \in I(v)} f((v, u)) = f(b_i) + f((b_i, b_{i \oplus_{13} 1})) = [2i] + [35 - 2i] = 35$ .

The graph  $C_3 \cup C_{4t+1}, t \geq 1$  is EVIMT with the magic constant 35.

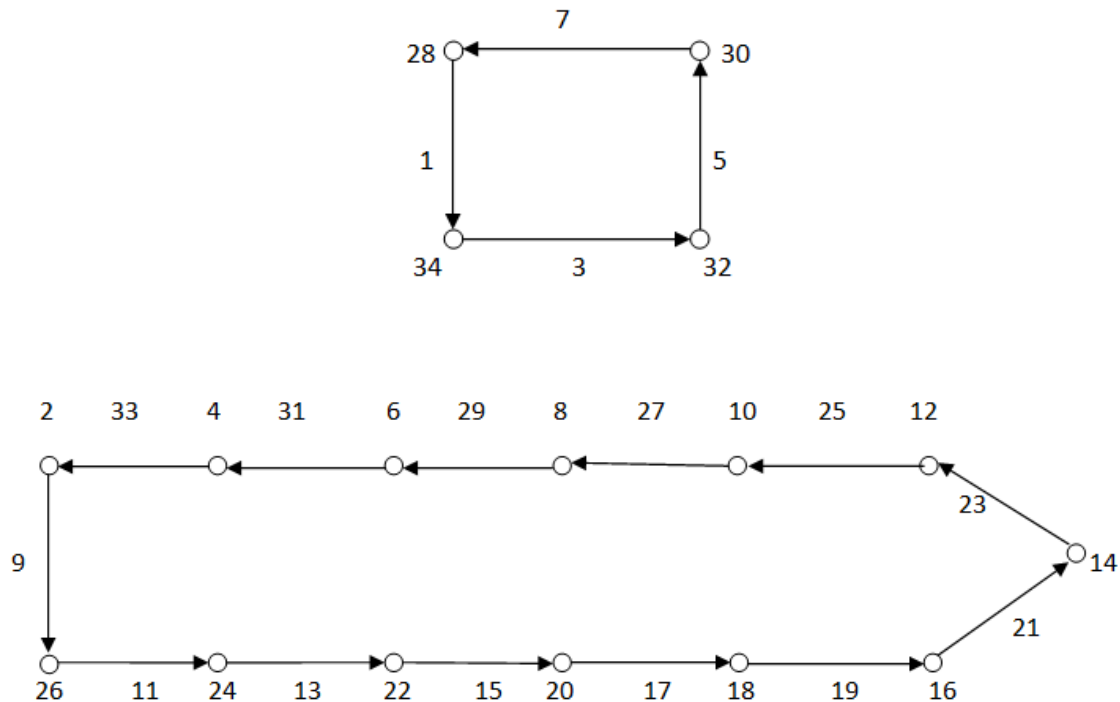


Figure 4:  $C_4 \cup C_{13}$   $k = 35$

### 3. CONCLUSIONS

In this paper we have discussed some cycles of graphs that admits EVIMTL. In future we can prove different types of graphs which satisfy EVIMTL.

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