
Using the Dynamic Programming Method to Determine the Optimal Decision

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Abstract: *The most important problem facing any administration is making the appropriate decision to obtain lower costs, so the aim of this research is to find the ideal strategy for establishing electric power stations in Iraq because of its great importance in all aspects of life, with the adoption of one of the branches of the task of applied mathematics, which is programming dynamism While researchers use it to find optimal solutions for various problems. The principles of dynamic programming and how to use them in solving the research problem were discussed and analyzed by dividing the completion time of the project into successive periods of time, and dividing the project into five phases to be contracted by the entity that processed the electrical units and all other requirements for the procurement phase of the project, and the second phase is Allocating appropriate places for the project, and the third stage is preparing suitable places for establishing basic buildings and warehouses suitable for the project. Implementation of the project, the stage of shipment, transfer of supplies to the warehouses designated for the project, and the final stage, completing the installation of electrical units and connecting them to the main national network, and all these stages include partial stages (according to the opposite pattern, respectively)*

Keywords: *Optimal Solution, Linear Programming, Back-To-Back Method, Methods and Processes, Strategy, Dynamic Programming.*

1. INTRODUCTION

Research's Problem

The dynamic programming has wide applications in various fields, with wide potential for finding optimal solutions for various problems in real life, for having the flexibility to determine the optimal decision in the production planning currency, as well as to use it to study optimal storage or to find the critical path for the design of genetic algorithms⁽⁷⁾. The study of



the distribution of investments between the economic sectors so the main problem of research is finding the best solution for the completion of the project through the study of the problem using one of the methods of operational research application dynamic programming method

Search Hypothesis:

Using one of the operational research methods to find the optimal strategy and the best solution for the construction of power plants in Iraq in a dynamic programming style with the adoption of reverse-style.

Research Goal

The goal of the research is to use dynamic programming method to determine the strategy and optimal solutions for the construction of power plants in Iraq to reduce the power crisis and optimize the utilization of available resources in order to achieve the goal in the least time and cost and how to overcome them through the establishment of electric power stations as well as laying the foundations for scientific investment in Future projects.

Research Style

The method of research is a study with a review of the theoretical concept with the expected results in the practical aspect through the dynamic programming method, with the adoption of the information documented by the Directorate of Electricity Distribution in Diwaniyah.

Field of Applied Study

Field: General Directorate of distribution of electricity Euphrates Middle / Directorate of electricity distribution Diwaniyah.

Time domain: Information was selected for the year (2017) for the implementation of a project in the Hakim region.

The Most Basic Concepts of Dynamic Programming:

Dynamic programming starts with a small part of the problem and tries to arrive at an optimal solution for this part and then gradually take another part of this matter and reach another model solution, taking into consideration the solution for the first part. and so on until the matter is resolved on the fullest picture and in all respects. The basic concepts of dynamic programming can be explained as follows:

1. Stage: represents the transition from one case to another or the period of time on which the main problem is divided into secondary problems.

2. State Variables: These are the variables that represent the connection between the previous stage and the current stage or the process of linking the current stage and the next stage. By determining the process of linking, the optimal decision is made for the current stage.

3. The Optimal Policy: Determination and value with the identification of digital relationships and values of the problem.

4. Develop the Optimal Yield Function, which allows calculating the optimal policy of the situation at each stage and then determining the optimal return function for the first stage.

5 - Create an Explanatory table showing the required values and calculations for each stage. Known that the stage is the part of the problem needed to make a decision the case is the (link) between the successive stages so that each decision is produced independently. The characteristics that characterize the dynamic programming from the quantitative analysis tools related to decision making are that they make decisions for the dynamic programming question of successive stages, which are segmented into (n) Decisions are made on each issue individually. The results of the decisions in DP are also based on a small number of variables for each stage. In addition, the decision at each stage has no effect on the number of variables on which the decision is based. , Successive relationships are used by associating the optimal policy of stage n with phase $n - 1$ according to the equation below:

$$F_n^* (S_n) = \text{Opt} [r_n(d_n) \times F_{n-1} (S_n \times d_n)]$$

Where (X) is any mathematical relationship between (S_n, d_n) so that the method of solution turns from one stage to another and at each stage we produce the best policy until the last stage.

6. Return Variable is the standard variables for measuring the total return at each stage. These variables are the decision function (d_i) and the case vector (x_i). This function can be expressed as follows:

$$R_i = r_i (x_i, d_i) \quad i = 1, 2, 3, \dots, n \quad \dots (1)$$

7. Transformation function is a mathematical expression that describes the relationship between different stages and contributes to the transfer of the optimal solution from the current phase (outputs) to a later stage (input) for the purpose of making the optimal decision as follows (4):

$$x_{i+1} = t_i (x_i, d_i) \quad \dots (2)$$

8. The Recursive Equation: The concept of repetitive equation is the basis for the formulation of any optimization problem through dynamic programming and is based on a repetitive method of arithmetic operations. This is the natural sequential equation of dynamic programming and at the same time reflects the basic principle of optimization to Bellman, (The ideal policy has a characteristic, whatever the initial case and the initial decision, the remaining decisions should be the best policy by reference to the situation resulting from the first decision).

As it contributes to the optimal solution for each stage independently and can calculate the optimal return of the previous stages so that we can reach the final solution optimization problem, when calculating the optimal return for the optimal (n) of the stages, it depends on the optimal return to ($n - 1$) From the previous stages with the addition of the optimal return of phase (n), which leads to the use of the formula to obtain the optimal solution for each stage



independently and through the equation we can calculate the total cumulative return of the previous stages and thus we find the best solution to the final problem, Restored Repeatability of each stage in the following form:

So the return function is the first stage

$$R_i = r_i(x_i, d_i) \dots \dots (3)$$

The optimum yield for the first stage represents a function for all decision variables in those stages and the best ones are selected

$$f_1(x_1) = \text{opt}\{r_1(x_1, d_1)\} \dots \dots \dots (4)$$

In the second stage we take the following formula:

$$f_2(x_2) = \text{opt}\{r_2(x_2, d_2) + f_1(x_1)\} \dots \dots (5)$$

And repeat the equation for all stages until we reach the final stage (Stage n), which is the following:

$$f_n(x_n) = \{r_n(x_n, d_n) + f_{n-1}(x_{n-1})\} \dots \dots (6)$$

Whereas:

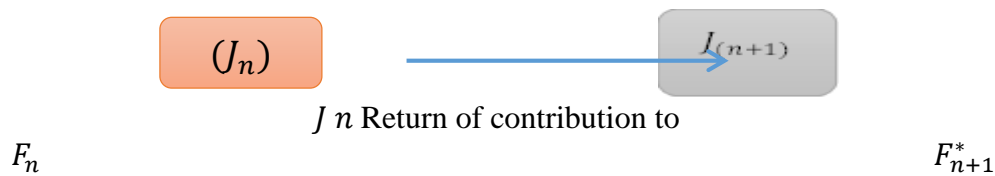
x_n : The state variable that can be assigned to the stage (n) is the resolution (dn) and the return function is $f_n(x_n)$ and the remainder of this variable is assigned to the stage ($n - 1$). The previous return function is $f_{n-1}(x_{n-1})$ Note that this situation is determined by $f_{n-2}(x_{n-2})$ and so on until we find the value of $f_1(x_1)$ which represents the return function of the initial phase.

Therefore, we can say that the return function depends on both the state variable and the decision in phase n and the optimal decision in phase n will be the decision that maximizes the yield or minimizes the value⁽²⁾ In addition to ensuring optimum utilization while improving performance by reducing the time and cost of projects in enterprises⁽³⁾.

Deterministic Dynamic Programming

This type is characterized by the(State)in the next phase (Next Stage) is determined by the state and decision in the current phase and not random variables and determines the return accurately and the results for each part of the problem is defined in other words that multiple decision process The stages are definite and deterministic, ie, the existence of a known output for each decision. This type is suitable for solving discrete problems, which contain a (Finite) number of solutions, for example the problems of allocation and sequence in the scheduling process.

In Figure (1) below shows the main structure of the specific dynamic programming
stage (n)
jn decision
Stage (n + 1)



In the figure above, note that in phase (n) the case (Jn) and resolution (jn) will move to In j_{n+1} phase status (n + 1)

Methods of Dynamic Programming Solution

The optimal solution is determined by the Forward method. The ideal policy starts from the first stage and the subsequent stages until the final stage (n_1, n_2, \dots, n_k) In the backward method, ($n_k, n_{k-1}, n_{k-2}, \dots, n_1$), so there are two ways to solve the problem:

Forward Computing Method: This method is based on the ascending order of the values of the functions of the order and uses the repetitive equation in the calculation of the value of the first function and progress the other functions until we reach the final function of the equation of the repetitive.

Backward Computation Method: This is the opposite method of the first method where the repetitive equation is used to calculate the function values in the order of the descending functions. In this method, we use the repetitive equation to calculate the values of the last stage and then step down to find the return values until we reach the first stage⁽²⁾. In this paper we will examine the method of background calculations (B.C)

Backward Computing Method

It is a way to find the best solution to the problem according to the dynamic programming method, which is the opposite way of application of the previous method, and uses the frequency relationship to find the best solution by moving from the back to forward phase stage and at each stage is the optimal plan for each of these cases until we reach the stage First, the order shall be descending as follows:

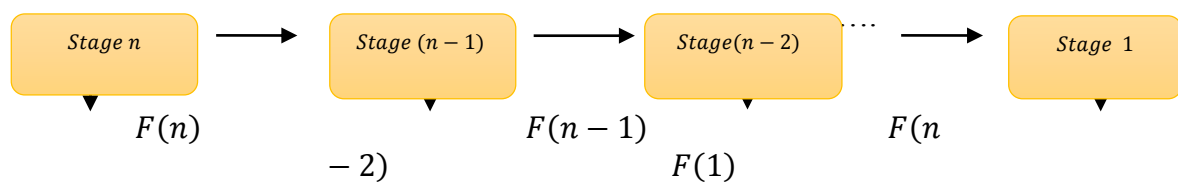


Figure (2) shows the method of background calculations for dynamic programming issues

The difference between the two methods is the method used to define the state of system. We also show that dynamic programming differs from linear programming as follows:

1-In solving linear programming problems, one standard model is used. In solving dynamic programming problems, no standard algorithm or model can be used to solve problems.

2 - In linear programming, the solution of the problem is divided into stages but does not give an acceptable solution (Feasible) for each stage while in solving the problems of dynamic programming, we deal with the problem by dividing them into sub-problems and find the best solution for each partial problem⁽¹⁾.

The Practical Side:

In order to solve the problem and find the best solution for the completion of the project by applying the method of the background calculations, the problem of research was described and fragmented from the time perspective, including the stages of completion of the project agencies:

- The stage of contracting with the equipped side and shipping the required supplies for the project.
- The stage of allocation of suitable places for the project.
- Configure these places with appropriate store construction.
- Implementation phase of the project (the process of setting up the generating units and connecting them to the national main network).

Dynamic programming was used to solve the project's network diagram starting with the start point and ending with the purpose of finding the longest time and determining the longest path between these two points and by dividing the project into stages as shown in Figure (1)

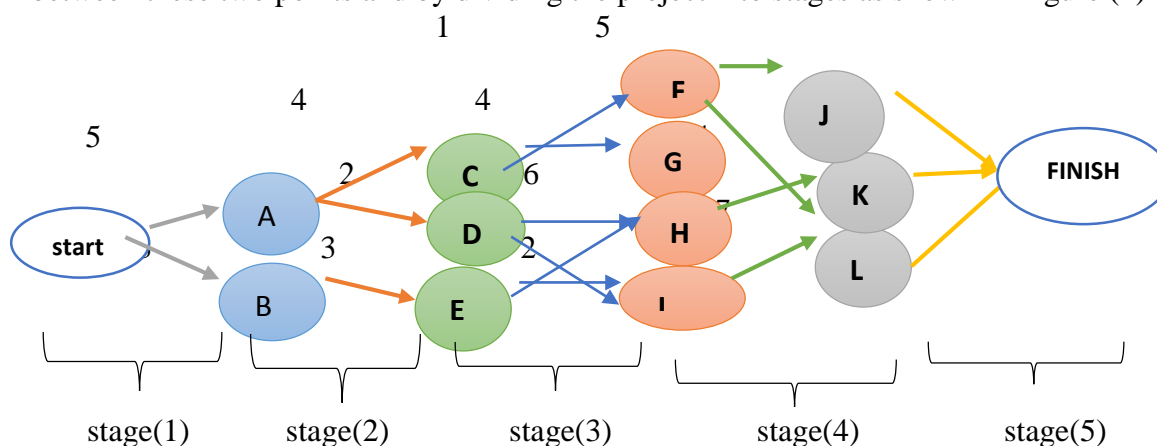


Figure (3) shows the fragmentation of the project into stages (preparation researcher)

Where the number of stages is (5) this means: N = 1,2,3,4,5

Each phase has a beginning and an end. This stage is the beginning of the next stage that will follow, and each stage has a number of cases that combine with the beginning of that stage. According to the implementation of the backward recursion, we will begin to solve the last phase (5). The equation used for this project is:

$$F_n (s_n , x_n) = t_s x_n + f^*_{n+1} (x_n)$$

$$f^*n (s_n) = \max \{ f_n (s , x_n) \} = f_n (s , x_n^*)$$

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$$\max \{ f_n (s, x_n) \} = t_n x_n + f_{n+1}^* (x_n)$$

wheras :

f_n^* : is the optimal access zone

x_n : the number of available stations

s_n : the number of stations to be allocated

For the purpose of applying the method of background calculations to solve the problem of dynamic programming, we will divide the problem into five stages ($n = 1,2,3,4,5$) as each phase involves a stage of implementation of production, and using the table below we begin to solve:

$$n = 5$$

s	$f_5^* (s)$	$x^* s$
J	5	FINISH
K	4	FINISH
L	7	FINISH

The optimum areas for this phase are (j, k, l)

Then we move on to the next stage of the solution, $n = 4$, in which the previous phase ($n = 5$) is used as the optimal stage according to the following equation:

$$\max \{ f_n (s, x_n) = t_n x_n + f_{n+1}^* (x_n) \}$$

The optimal function $F^*_s (s_n)$ will be used in the preceding phases $n = 4$

$X_4 \backslash S$	$f_4 (s, x_4) = (t_4, x_4) + f^*_5 (x_4)$			$f^*_4 (X_4)$	X^*_4
	J	K	L		
F	6	-	-	6	J
G	-	7	-	7	K
H	-	9	-	9	K
I	-	-	8	8	L

(J, K, K, L) are the optimal areas for phase IV.



After finding the optimal solution for the stage $n = 4$ we will find the optimal solution for the stage $n = 3$ depending on the optimal gel for the next stage and using the following equation:

$$\max \{ f_n(s, x_n) = t_n x_n + f^*_{n+1}(x_n) \}$$

$$n = 3$$

X_3 S	$f_3(s_3, x_3) = (t_3, x_3) + f^*_4(s_3)$				$f^*_3(X_3)$	X^*_3
	F	G	H	I		
C	10	12	-	-	12	G
D	-	-	12	11	12	H
E	-	-	13	12	13	H

When $n = 3$, we consider the optimal stages in this case to be the transition to (H, H, G) . After selecting the optimal regions, we move to stage $n = 2$

$$n = 2$$

X_2 S	$f_2(s, x_2) = (t_2, x_2) + f^*_3(X_2)$			$f^*_2(S_2)$	X^*_2
	C	D	E		
A	17	17	-	17	$C \text{ OR } D$
B	-	-	16	16	E

Where the methods of ~~A~~ C or A D and B E is the best route for distribution, and after the completion of the accounts related to stage $n = 2$ we move to the stage $n = 1$

$$n = 1$$

X_1 S	$f_1(s, x_1) = (t_1, x_1) * f^*_2(X_1)$		$f^*_1(S_1)$	X^*_1
	A	B		
START	17	16	17	A

From the results of the above stages, we deduce the optimal strategy:

START → A → ~~D~~ → ~~H~~ → K → FINISH

In this way we have found the critical path (the best solution), and then we have reached a solution to the problem of research.

These results lead us to say that the best strategy is

START → A → D → H → K → FINISH

Which were reached in the light of partial strategies

3.2 Conclusions and Summary of Accounts It is clear from the application of dynamic programming solutions to the problem of our research that the best solution to the problem is to adopt the critical path of the longer time required by the project to complete the construction of the electric power units, as shown in Fig. 2 that the longest path is equivalent to 17 units of time, Obtain two critical paths through which the best solution (time of project implementation) can be reached:

C . P (1) = START → A → C → G → K → FINISH = 17

C . P (2) = START → A → D → H → K → FINISH = 17

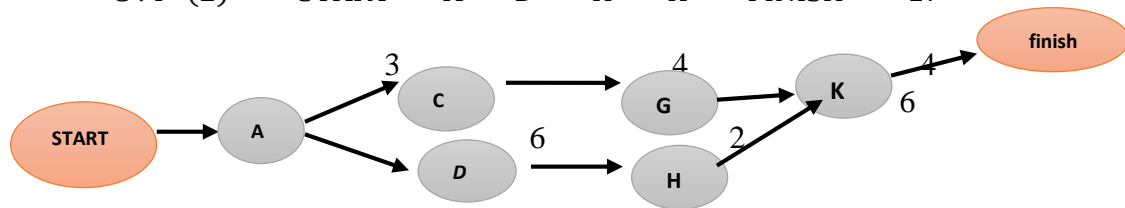


Figure (4) shows the critical path of the project (prepared by the researcher)

Through these two critical tracks, the project can be implemented as planned within the time frame allocated to each activity

2. CONCLUSIONS

1. Using dynamic programming in solving the problem of completion of projects leads us to good results in the identification of the shortest possible time to achieve.
2. The adoption of the scientific method refers to the efficiency of the plan where the results of the planning that this method of modern methods and efficient in reaching the optimal solution through the adoption of appropriate technical decisions.
3. The results achieved and through the possibilities available to the project shows that the optimal time to complete the project is (17) units of time and the critical path is through Table (2) This proves the achievement of the main objective,
4. Draw the attention of officials to the importance of strategic planning in the decision-making is optimized based on dynamic programming, which is to study all possible strategies for the project and choose the strategy that we reached the appropriate decision
5. To show the importance of the basic role of modern mathematics and its applications in finding solutions to economic problems and in achieving the main objective of the project.



Recommendations

1. Using the method (CPM) for its simplicity, along with the identification of responsibilities for the implementation of the project clearly and accurately of the results with the preparation of completion reports for each activity and identify the obstacles, which avoids the possibility of loss and delays in the implementation of projects.
2. Researchers used this method in their research for accuracy and ease of application in solving problems.
3. The use of the network diagram and dynamic programming in the study of other problems related to the subject.
4. Attention to the development of modern methods that solve problems and address the obstacles facing the implementation of economic projects and productivity and using methods such as linear programming, PERT and other methods of solving problems to compare the results.
5. The application of dynamic programming in the completion of the project in Diwaniyah Directorate of the distribution would lead to the optimal solution to the achievement with the need to adopt the method in the future projects of the country.

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