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Strength and Limitation of Biological Paradigm during Pandemic of Covid-19: An Application of Queueing Theory

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***Abstract:** The coronavirus disorder is an epidemic induced by the SARS-CoV-2 germ. The health region is recognized as one of the most important areas in the service sector due to its direct connection to human well-being, particularly in context of the pandemic COVID-19. The overcrowding of patients in the patient registration department, outpatient department, and pharmacy department is the most important issue facing hospitals these days. The investigation of the patient registration, outpatient, and pharmacy departments of PIMS hospital in Islamabad and Benazir Bhutto hospital in Rawalpindi served as the foundation for this study. Queuing Theory is an extension of operation research and is concerned with the aspect of queues. Additionally, the multiple-server queueing model is used to calculate the performance parameters. The approach of queuing simulation used in this research deals with multiple queues and multiple server models. Poisson distribution is used to estimate the arrival rate and exponential distribution is used to calculate the service rate. The satisfaction level of patients with hospital services during the pandemic (Omicron duration) is measured through the questionnaire. As compared to other departments, the utilization factor is higher in the pharmacy department in both hospitals. From PIMS hospital 44% of patients are satisfied with the service system while from BBH 40% of patients are satisfied with the service system.*

***Keywords:** SARS-CoV-2, Queueing Theory, Queueing Simulation, Poisson Distribution, Exponential Distribution.*

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1. INTRODUCTION

The coronavirus epidemic began in Wuhan, China, in December 2019 as pneumonia of unidentified origin that is speedily spreading to other countries. The World Health Organization confirmed the coronavirus epidemic as the sixth public health emergency of international concern on 30 January 2020, and on 11 March 2020, declared as an epidemic. International Committee on Taxonomy of Viruses called the germ SARS-CoV-2 and syndrome COVID-19. Researchers detected that the severe simple respiratory syndrome coronavirus 2 manifests itself in aerosols for up and around three hours, up to 24 hours in wood, up to four hours in copper, and up to two to three days in plastic and stainless steel. These effects specify basic evidence for the persistence of SARS-CoV-2, which causes the syndrome COVID-19, and recommend that an individual can receive the germ through the air and later touch dirty kinds of stuff [7]. The World Health Organization (WHO) was swiftly notified of the novel SARS-CoV-2 (B.1.1.529) variant's identification in Botswana and South Africa in November 2021 as a result of genomic surveillance. On December 13, 2021, Pakistan reported the first Omicron case, and since then, COVID-19 cases have increased [11].

The paradigm employs the term "pathogenic." Sickness is defined as a divergence from the body's normal biological functioning, and health is defined as the absence of disease. The paradigm is named "biomedical" because it necessitates highly qualified scientific experts, with a team of other specialists including laboratory technicians, pharmacologists, biochemists, nutritionists, nurses, and others supporting the doctor. Health care is primarily administered in this setting, the model is referred to as "hospital-based." Such services necessitate the use of complex and costly instruments as well as high-tech equipment. Hospitals can better utilize both human and technology resources by keeping patients together [3].

Particularly in the wake of natural or man-made disasters, hospitals are essential for providing primary healthcare to people. As a result of pandemics, disease transmission may spread more widely, and patient demand may fluctuate and increase quickly, putting hospital capacity and the efficiency of the entire healthcare system at risk. Hospitals must be ready to handle the task of such an epidemic disaster by the time these occurrences take place. Currently, the vast majority of the world is battling an epidemic known as COVID-19. The world is experiencing a surge in the number of infected people who are arriving at hospitals in large numbers and regularly [5].

Queueing Theory

Operations Research (O.R.) is a discipline that offers scientific approaches for resolving issues in real life and aids in figuring out how to make the greatest use of scarce resources. With the development of a team of scientists to investigate the strategic and tactical issues associated with various military operations, operations research was created and rose to popularity during World War II in Britain. In the 1930s, the field of operations research was originally acknowledged as a science and subfield of applied mathematics. Numerous analytical techniques are used in this area however, queuing theory is the most popular and widely used analytical method in several fields of life. Queuing Theory has a nearly 100-year history. In the

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early 1920s, A. K. Erlang, a Danish telephonic engineer, invented queuing theory. Erlang was concerned with the capacity and usage of the equipment and lines when researching automatic telephone switching applications. Queuing Theory was widely applicable to a variety of situations by the end of World War II. The study of waiting lines is known as queuing theory. The issue of waiting lines can be solved using one of two basic strategies. At first, a performance measure is calculated using certain queuing models, and the arrival rate and service rate values are established. Simulation is the second way for resolving queuing models. Simulation is utilized for queuing models that are very challenging or impossible to solve using analytical techniques. To analyze a real queuing system, a queuing model is utilized as a close approximation. There are several different kinds of queuing models, but the two that are most frequently used are single-server and multiple-server queuing models [9].

The foundation of queue analysis is the act of creating a mathematical model that depicts the arrival of an item that joins the queue, the criteria by which they are permitted into service, and the amount of time it takes to serve. When items arrive at a location that provides the service they require, they form a line and are referred to as being in a "queue," which is a waiting line. The term "arrival" refers to the typical amount of consumers who need assistance within a specific time range. "Customers" can be any modelable thing that must wait for a process to happen, including individuals, work-in-progress records, raw materials, incoming digital messages, and more. The size of the queue can be unlimited or finite. "Server" can refer to a person, a machine, or any other entity that executes a task for customers who are waiting [10].

Figure 1: Basic Structure of Queuing System

Problem Statement

The present research is based upon the evaluation of the queueing analysis of the patient registration department, outpatient department (OPD), and pharmacy department of public hospitals in Rawalpindi and Islamabad during the pandemic COVID-19. As a result of the pandemic, disease transmission may spread more widely, and patient demand may fluctuate and increase quickly, putting hospital capacity and the efficiency of the entire healthcare system at risk. Patients must wait before being served. Due to lengthy waits in the OPD waiting room and significantly increased workloads, subjects and doctors alike are stressed and anxious. The patients' registration, pharmacy, and OPD's current queuing mechanism need to be evaluated.

Objectives

The research has the following objectives.

1. The innovative queuing models are analyzed and simulated.
2. To calculate the parameters of queuing models.

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3. To find which queuing model reduces the waiting time in queues.
4. To evaluate the effectiveness of the current queue mechanism.

Literature Review

In the light of queuing theory, various studies have been presented to examine its components and models. A superior number of research has been presented globally to evaluate the models of queuing theory.

Zhao et al. [13] Identified that Patients' wait times have been caused by a number of factors, including a shortage of beds, doctors, and nurses, delays between doctor referrals for admission and patient transfer to assigned beds, and ineffective lab and radiology test procedures. Yousefi et al. [12] created a framework for agent-based modelling to understand the pattern of patients who leave the hospital's emergency room without being examined by a doctor. To accomplish the goal of the study, computer modelling and cellular automata approaches were employed. The results of the data were validated using a real-world case study, and it was determined that primarily four factors had influenced the results. Hu et al. [6] investigated the utility of queuing supposition in eradicating emergency department issues. The study was done to enable emergency departments to effectively simulate patient behavior in order to reduce the limitations of their applications. Different discrete approaches were employed to validate a single issue increasing the effectiveness of this study. Bai et al. [2] analyzed studies done on intensive care units by using queuing theory. The primary goal of the study was to understand the value of operation research and quality management in solving issues pertaining to the healthcare industry. Different issues, such as decision horizons and the design of various scenarios in accordance with demand and supply strategy, were addressed and resolved. In addition, gaps in research and prospects for future outcomes were noted. Mustafa & Nisa [9] examined two models of queuing systems: the first is the M / M / I single-server queue model and the second is the M / M / C multiple-server queue model. They also used queuing simulation in their study. Girija et al. [4] presented a basic probabilistic model to standardize mutual behavior in order to estimate how long the COVID-19 medication cycle will recognize and classify people already tested and negative or positive overall results. They showed that the patient waiting time in the order of consultation, detection or cure of COVID-19 in case of inequalities within the organization improves globally according to the logarithm rule. Zimmerman et al. [14] performed a queueing model to report ventilation capacity management in different COVID-19 epidemic scenarios. The model was used to organize ventilation capacity during the first wave of COVID-19 in British Columbia, Canada.

2. MATERIALS AND METHODS

This section illuminated one of the biological paradigms through the use of queueing theory in hospitals during the pandemic COVID-19. Major public hospital in Islamabad and Rawalpindi served as the study's subject. Our study focused primarily on the outpatient, pharmacy, and patient registration departments of PIMS (Pakistan Institute of Medical Sciences) hospital Islamabad and Benazir Bhutto hospital Rawalpindi. The effectiveness of the queuing mechanism in these departments was evaluated using a queuing analysis technique. When the queueing process is complex and queuing analysis becomes imprecise, queuing simulation was

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utilized to measure performance or efficiency of a system more precisely. Multiple queues and multiple server queueing model is the fundamental queueing model used for queueing analysis and simulation in these departments. These are the following basic assumptions for queueing models:

1. Patient's arrival rate with parameter λ follows poison process.
2. The inter-arrival time of the poison process was exponentially and independently distributed.
3. The patients' service times were distributed exponentially with the parameter.
4. The interval between patient arrival times was independent and exponentially distributed.
5. There was an endless line.
6. Every patient received the services they wanted.
7. First-come, first-served service discipline is implemented [1].

2.1 Patient's Registration Department
The fundamental queueing model that this department have seen is multiple-queue multiple-servers, or M/M/s, where M/M stands for the Poisson probability distribution of arrivals and departures and s stands for number of servers. Data for this investigation are only gathered for 60 minutes, from 8:20:00 to 9:20:00, each morning. Both queuing analysis and queuing simulation were used to analyze this model. Considering four servers i.e. s= 4 in multiple server queuing model with multiple-queues for queuing analysis and queueing model labeled as M/M/4.

2.2 Outpatients Department (OPD)
Multiple queues with multiple servers were used in this department queuing process, and this model's abbreviation is M/M/s, where M/M stands for the Poisson probability distribution of arrivals and departures and denotes the number of servers. This research only included observations from 9:45 am to 10:45 am from medicine OPD. Both queuing analysis and queuing simulation were used to analyze this model. Considering four servers i.e. s= 4 in multiple server queuing model with multiple-queues for queuing analysis and queuing model labeled as M/M/4.

2.3 Pharmacy Department
Multiple queues with multiple servers were used in this department's queuing process, and this model's abbreviation is M/M/s, where M/M stands for the Poisson probability distribution of arrivals and departures and s denotes the number of servers. The pharmacy department of a public hospital is only examined for 30 minutes as part of this study, from 11:00 AM to 11:30 AM. Both queuing analysis and queuing simulation were used to analyze this model. Considering four servers i.e. s= 2 in multiple server queuing model with multiple-queues for queuing analysis and queuing model labeled as M/M/2.

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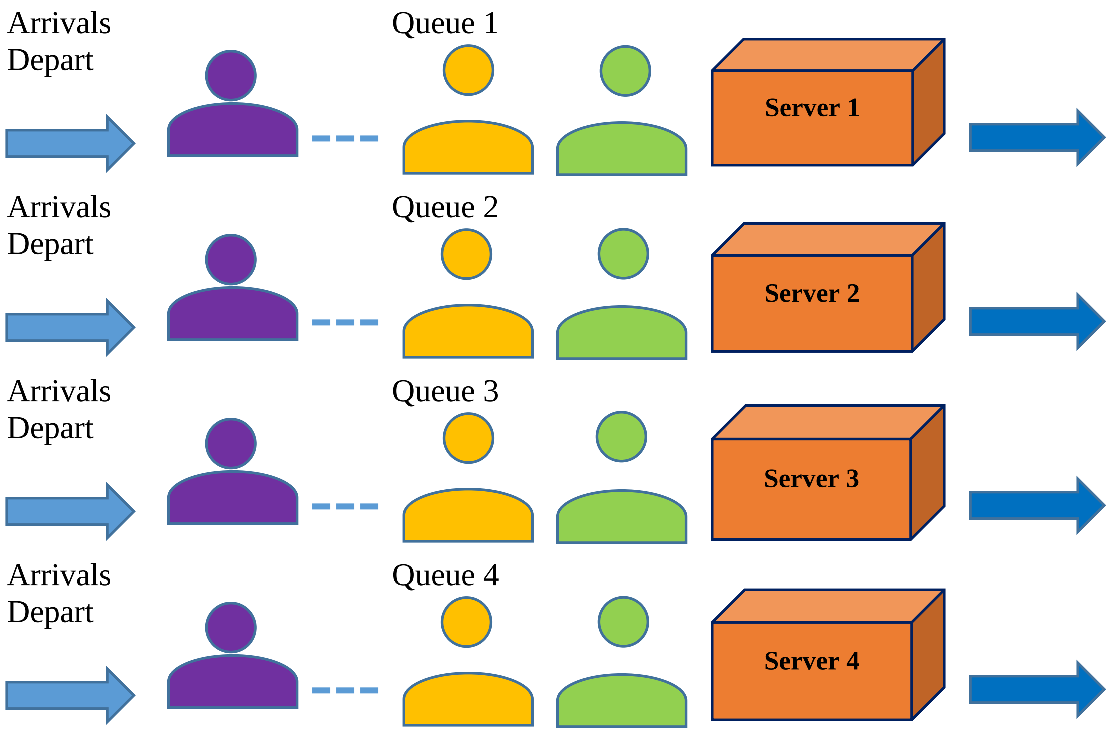

Figure 2: Queuing Model (multiple queues with multiple servers)

The queuing performance measures for the hospital are mentioned below:

For M/M/C Model

$$p_n = \frac{\lambda^n}{c!\mu^n c^{n-c}} p_o \quad , \quad w_s = \frac{L_s}{\lambda}$$

$$w_q = \frac{L_q}{\lambda} \quad , \quad \rho = \frac{\lambda}{C\mu}$$

$$L_s = \rho + L_q \quad , \quad L_q = \frac{\rho^{c+1}}{(c-1)!(c-p)^2}$$

Waiting probability of a customer is = $p_0 = \dfrac{1}{\sum \dfrac{\rho n}{n!} + \dfrac{\rho c}{c!}(1-\dfrac{\rho}{c})}$

Mean count of the idle servers = n – (Mean no of customer served)

Efficiency of M/M/C model = $\dfrac{\text{(Average number of customer served)}}{\text{(Total number of customers served)}}$

Where the parameters are as follows
n = total count of patients in the system , s = number of servers

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λ = Rate of Arrival ,
μ = Rate of Service
ρ = Utilization factor ,
L_q = Mean number of patients in line
L_s = Mean number of patients in system ,
W_q = Mean down time spent for facility in queue
W_s = Mean down time spent in system ,
p_0 = The percentage time each counter is busy
p_n = The probability that exactly n clients are in the structure

2.4 Method of Data Collection

The research data was collected from PIMS hospital in Islamabad and Benazir Bhutto hospital Rawalpindi, Four days a week, for a period of two weeks. In the outpatient section, data were solely gathered from patients and doctors, not from any other medical personnel. Data gathering was primary. The method used for data collection are, “questionnaire”, “Direct observation” and “Interviews”.

3. RESULTS AND DISCUSSIONS

We have utilized WinQsb software to compute the performance measures utilizing the arrival rate, service rate, and the number of servers in public hospitals using queuing analysis and simulation of single server and multiple server models.

3.1 Results of Patient’s Registration Department from both Hospitals

Table 1: Parameters and performance measures of M/M/4 queueing model using queueing formulas of patient’s registration department from PIMS hospital

Parameters and performance measures	Symbols	Monday	Tuesday	Wednesday	Thursday
Number of servers	c	4	4	4	4
Rate of Arrival	λ	250 / hour	266 / hour	279 / hour	176 / hour
Rate of Service	μ	178 / hour	176 / hour	234 / hour	163 / hour
Overall system utilization	$\rho = \lambda / c\mu$	35. 1124%	37.7841%	29.8077%	26.9939%
Probability that all servers are idle	P_o	24.3759%	21.8422%	30.2520%	33.8974%
Mean number of patients in the system / hour	L_s	1.4375	1.5577	1.2077	1.0895
Mean number of patients in the queue / hour	L_q	0.0330	0.0464	0.0154	0.0097
Mean time patients spends in the system / hour	W_s	0.0057	0.0059	0.0043	0.0062

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Mean time patients spends in the queue / hour	W_q	0.0001	0.0002	0.0001	0.0001

Table 2: Parameters and performance measures of M/M/4 queueing model using queueing simulation of patient's registration department from PIMS hospital

Parameters and performance measures	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers (c)	4	4	4	4
Overall system utilization (ρ)	35. 3586%	37.9984%	30.0938%	26.9611%
Probability that all servers are idle	24.3313%	21.7127%	29.9444%	34.0923%
Mean number of patients in the system (L_s) / hour	1.4516	1.5660	1.2216	1.0897
Mean number of patients in the line (L_q) / hour	0.0373	0.0461	0.0178	0.0113
Mean time patients spends in the system (W_s) / hour	0.0058	0.0059	0.0044	0.0062
Mean time patients spends in the line(W_q) / hour	0.0001	0.0002	0.0001	0.0001
Number of simulations	1000	1000	1000	1000

Table 3: Parameters and performance measures of M/M/4 queueing model using queueing formulas of patient's registration department from Benazir Bhutto hospital

Parameters and performance measures	**Symbols**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers	c	4	4	4	4
Rate of Arrival	λ	233 / hour	280 / hour	265 / hour	241 / hour
Rate of Service	μ	174/ hour	188 / hour	203 / hour	181 / hour
Overall system utilization	$\rho = \lambda / c\mu$	33.4770%	37.2340%	32.6355%	33.2873%
Probability that all servers are idle	P_o	26.0604%	22.3428%	26.9697%	26.2628%
Mean number of patients in the system / hour	L_s	1.3655	1.5327	1.3289	1.3572
Mean number of patients in the line / hour	L_q	0.0264	0.0433	0.0235	0.0257
Meantime patients spends in the system / hour	W_s	0.0059	0.0055	0.0050	0.0056
Mean time patients spends in the line / hour	W_q	0.0001	0.0002	0.0001	0.0001

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Table 4: Parameters and performance measures of M/M/4 queueing model using queueing simulation of patient's registration department from Benazir Bhutto hospital

Parameters and Performance measures	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers (c)	4	4	4	4
Overall system utilization (ρ)	33.4918%	37.3604%	32.7489%	33.1617%
Probability that all servers are idle	26.1798%	22.2685%	26.8992%	26.4434%
Mean count of patients in the system (L_s) / hour	1.3675	1.5436	1.3355	1.3532
Mean count of patients in the line (L_q) / hour	0.0278	0.0492	0.0256	0.0267
Mean time patients spends in the system (W_s) / hour	0.0059	0.0055	0.0050	0.0056
Mean time patients spends in the line (W_q) / hour	0.0001	0.0002	0.0001	0.0001
Number of simulations	1000	1000	1000	1000

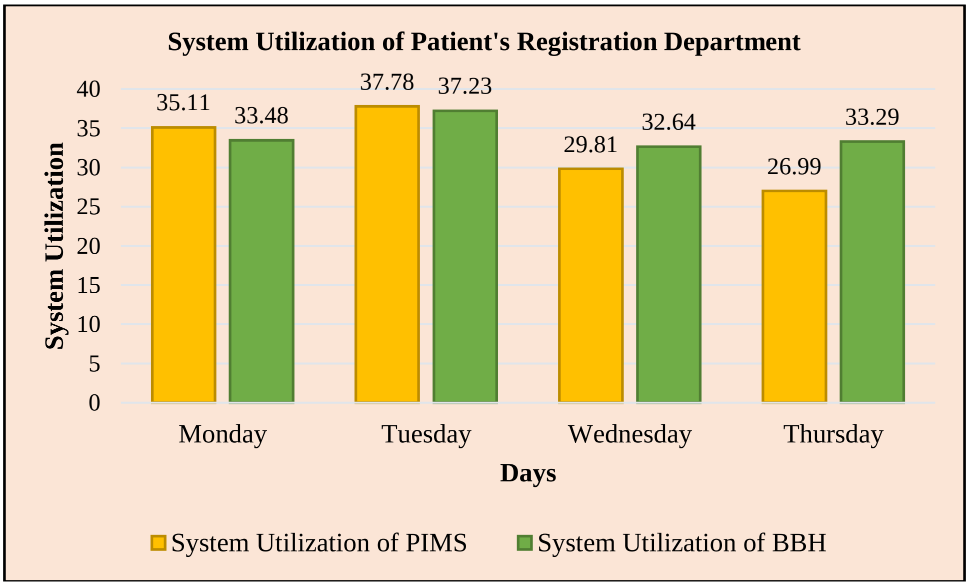

Figure 3: Comparison of Utilization Factor at Patient's Registration Department

3.2 Results of Outpatient's Department from both Hospitals

Table 5: Parameters and performance measures of M/M/5 queueing model using queueing formulas of outpatient's department from PIMS hospital

Parameters and performance measures	**Symbols**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers	c	5	5	5	5
Rate of Arrival	λ	86 / hour	86 / hour	70 / hour	80 / hour
Rate of Service	μ	66 / hour	76 / hour	52 / hour	68 / hour

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Overall system utilization	$\rho = \lambda / c\mu$	26.0606%	22.6316%	26.9231%	23.5294%
Probability that all servers are idle	P_o	27.1506%	32.2413%	26.0009%	30.8235%
Mean number of patients in the system / hour	L_s	1.3071	1.1335	1.3510	1.1788
Mean number of patients in the line / hour	L_q	0.0041	0.0019	0.0048	0.0023
Mean time patients spends in the system / hour	W_s	0.0152	0.0132	0.0193	0.0147
Mean time patients spends in the line / hour	W_q	0.0000	0.0000	0.0001	0.0000

Table 6: Parameters and performance measures of M/M/5 queueing model using queueing simulation of outpatient's department from PIMS hospital

Parameters and performance measures	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers (c)	5	5	5	5
Overall system utilization (ρ)	26.0538%	22.5350%	27.7222%	23.5705%
Probability that all servers are idle	27.2417%	32.5371%	26.3500%	30.7840%
Mean number of patients in the system (L_s) / hour	1.3066	1.1285	1.3404	1.1805
Mean number of patients in the line (L_q) / hour	0.0039	0.0017	0.0043	0.0020
Mean time patients spends in the system (W_s) / hour	0.0152	0.0132	0.0193	0.0148
Mean time patients spends in the line (W_q) / hour	0.0000	0.0000	0.0001	0.0000
Number of simulations	1000	1000	1000	1000

Table 7: Parameters and performance measures of M/M/4 queueing model using queueing formulas of outpatient's department from Benazir Bhutto hospital

Parameters and performance measures	**Symbols**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers	c	4	4	4	4
Rate of Arrival	λ	84 / hour	103 / hour	114 / hour	73 / hour
Rate of Service	μ	72 / hour	93 / hour	107 / hour	65 / hour
Overall system utilization	$\rho = \lambda / c\mu$	29.1667%	27.6882%	26.6355%	28.0769%
Probability that all servers are idle	P_o	31.0476%	32.9604%	34.3912%	32.4467%

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Mean number of patients in the system / hour	L_s	1.1806	1.1185	1.0746	1.1348
Mean number of patients in the line / hour	L_q	0.0139	0.0109	0.0091	0.0117
Mean time patients spends in the system / hour	W_s	0.0141	0.0109	0.0094	0.0155
Mean time patients spends in the line / hour	W_q	0.0002	0.0001	0.0001	0.0002

Table 8: Parameters and performance measures of M/M/4 queueing model using queueing simulation of outpatient's department from Benazir Bhutto hospital

Parameters and Performance measures	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers (c)	4	4	4	4
Overall system utilization (ρ)	29.5858%	27.6385%	26.7583%	28.2600%
Probability that all servers are idle	30.6299%	33.1393%	34.2606%	32.1542%
Mean number of patients in the system (L_s) / hour	1.1997	1.1178	1.0815	1.1442
Mean number of patients in the line (L_q) / hour	0.0163	0.0123	0.0112	0.0138
Mean time patients spends in the system (W_s) / hour	0.0143	0.0109	0.0095	0.0157
Mean time patients spends in the line (W_q) / hour	0.0002	0.0001	0.0001	0.0002
Number of simulations	1000	1000	1000	1000

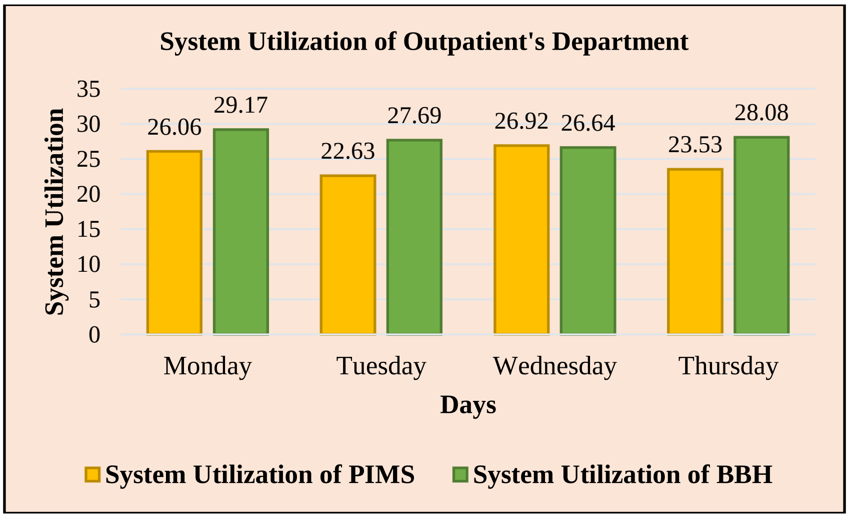

Figure 4: Comparison of Utilization Factor at Outpatient's Department

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3.3 Results of Pharmacy Department from both Hospitals

Table 9: Parameters and performance measures of M/M/2 queueing model using queueing formulas of pharmacy department from PIMS hospital

Parameters and performance measures	Symbols	Monday	Tuesday	Wednesday	Thursday
Number of servers	c	2	2	2	2
Rate of Arrival	λ	99 / hour	112 / hour	84 / hour	70 / hour
Rate of Service	μ	93 / hour	104 / hour	79 / hour	64 / hour
Overall system utilization	$\rho = \lambda / c\mu$	53.2258%	53.8462%	53.1646%	54.6875%
Probability that all servers are idle	P_o	30.5263%	30.0000%	30.5785%	29.2929%
Mean number of patients in the system / hour	L_s	1.4853	1.5167	1.4822	1.5604
Mean number of patients in the line / hour	L_q	0.4208	0.4397	0.4190	0.4667
Mean time patients spends in the system / hour	W_s	0.0150	0.0135	0.0176	0.0223
Mean time patients spends in the line / hour	W_q	0.0043	0.0039	0.0050	0.0067

Table 10: Parameters and performance measures of M/M/2 queueing model using queueing simulation of pharmacy department from PIMS hospital

Parameters and performance measures	Monday	Tuesday	Wednesday	Thursday
Number of servers (c)	2	2	2	2
Overall system utilization (ρ)	52.9263%	53.6140%	52.7670%	54.8977%
Probability that all servers are idle	30.7858%	30.2348%	30.8730%	29.1372%
Mean number of patients in the system (L_s) / hour	1.4721	1.5109	1.4933	1.6034
Mean number of patients in the line (L_q) / hour	0.4135	0.4386	0.4379	0.5055
Mean time patients spends in the system (W_s) / hour	0.0149	0.0136	0.0179	0.0229
Mean time patients spends in the line (W_q) / hour	0.0042	0.0039	0.0053	0.0072
Number of simulations	1000	1000	1000	1000

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Table 11: Parameters and performance measures of M/M/2 queueing model using queueing formulas of pharmacy department from Benazir Bhutto hospital

Parameters and performance measures	**Symbols**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers	c	2	2	2	2
Rate of Arrival	λ	117 / hour	97 / hour	137 / hour	101 / hour
Rate of Service	μ	102 / hour	85 / hour	129 / hour	95 / hour
Overall system utilization	$\rho = \lambda / c\mu$	57.3529%	57.0588%	53.1008%	53.1579%
Probability that all servers are idle	P_o	27.1028%	27.3408%	30.6329%	30.5842%
Mean number of patients in the system / 30 minutes	L_s	1.7093	1.6921	1.4791	1.4819
Mean number of patients in the line / 30 minutes	L_q	0.5623	0.5509	0.4171	0.4188
Mean time patients spends in the system / 30 minutes	W_s	0.0146	0.0174	0.0108	0.0147
Mean time patients spends in the line / 30 minutes	W_q	0.0048	0.0057	0.0030	0.0041

Table 12: Parameters and performance measures of M/M/2 queueing model using queueing simulation of pharmacy department from Benazir Bhutto hospital

Parameters and Performance measures	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
Number of servers (c)	2	2	2	2
Overall system utilization (ρ)	57.4414%	56.7133%	52.7894%	52.6292%
Probability that all servers are idle	26.8603%	27.6161%	30.9336%	31.1133%
Mean number of patients in the system (L_s) / 30 minutes	1.7362	1.6737	1.4878	1.4641
Mean number of patients in the line (L_q) / 30 minutes	0.5874	0.5395	0.4320	0.4115
Mean time patients spends in the system (W_s) / 30 minutes	0.0148	0.0174	0.0109	0.0146
Mean time patients spends in the line (W_q) / 30 minutes	0.0050	0.0056	0.0032	0.0041
Number of simulations	1000	1000	1000	1000

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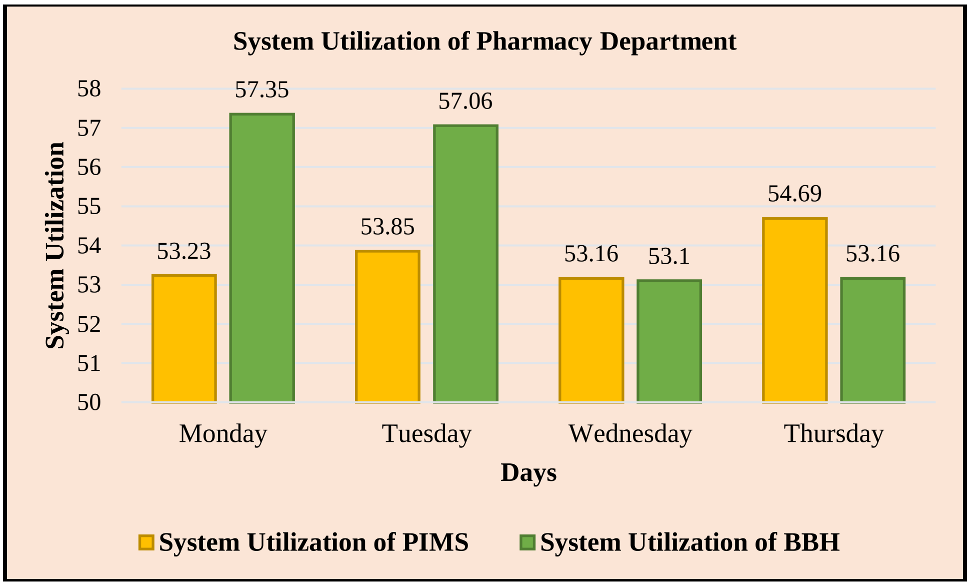

Figure 5: Comparison of Utilization Factor at Pharmacy Department

3.4 Questionnaire Analysis

The questionnaire data was collected from two distinct hospitals in Rawalpindi and Islamabad to assess the degree of patient satisfaction in each facility. A sample of 150 questionnaires was taken from hospitals; 52 patients responded in BBH, 48 responded in PIMS, and 50 individuals did not respond. Questionnaire analysis is done by using the Cross tabulation and frequency method in SPSS software. From PIMS hospital 44% patients are satisfied with service system while from BBH 40% patients are satisfied with service system from overall sample selected. According to 65% respondents in PIMS hospital and 79% respondents from BBH, COVID vaccination card is necessary for registration during pandemic. In PIMS hospital 48% of patients are satisfied with SOP's of Covid-19, and 50% respondents from BBH are satisfied with SOP's. We used SPSS software to measure the strength of relationship between Age group and satisfaction level of patients.

Table 13: Strength of relationship between Age and Satisfaction Level of Patients

	Factors		**Age of Patients**	**Satisfaction Level of Patients**
Spearman's rho	Age of Patients	Correlation Coefficient	1.000	.047
		Sig. (2-tailed)	.	.643
		N	100	100
	Satisfaction Level of Patients	Correlation Coefficient	.047	1.000
		Sig. (2-tailed)	.643	.
		N	100	100

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4. CONCLUSION

The queuing characteristics at two public hospitals of Rawalpindi and Islamabad are analyzed using queueing analysis and queuing simulation in three departments. Multiple server queuing models have been used for these analysis. The queueing simulation gives better results as compare to queueing analysis. The comparison is built on the basis of the utilization factor of each selected department at both hospitals. On Tuesday, the utilization factor in both hospitals' patient registration departments is higher. In PIMS hospital utilization factor is higher on Wednesday and in BBH utilization is higher on Monday in OPD. In PIMS hospital utilization factor is higher on Thursday and in BBH utilization is higher on Monday in the Pharmacy department. It is concluded that, the waiting time of patients in each department of PIMS Hospital is less than that of BBH. The results showed that, for both hospitals, the outpatients department had a lower patient wait time than the other departments due to the presence of more servers in OPD than in the other departments. And the pharmacy section has a longer line of waiting patients. Strength of relationship between Age and Satisfaction level is 0.047 which shows that they are positively correlated.

Recommendations

The availability of entertainment means such as reading materials or an Internet facility may improve the anticipation of the patient's wait time and their happiness. The full utilization of electronic media and other follow-up communications (messaging platforms) can be used to decrease the number of unwarranted and time-wasting face-to-face appointments. Hospitals should increase the number of servers and appoint hard-working staff. In order to improve public healthcare, this research study was only conducted at the medical OPD, but it might also be conducted at all hospital OPD’s. It is also possible to conduct a seasonal analysis of the affiliated OPD's queuing system.

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