



On Supra iR -Open Set in Topological Spaces and Some Applications

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Received: 27 July 2023

Accepted: 13 October 2023

Published: 01 December 2023

Abstract: *In this work we introduce the supra iR -open set in supra topological spaces. , a novel class of supra i -open sets. This class of sets exactly lies between supra regular open and supra i - open sets. We also look at its basic characteristics and compare it to other sorts of data supra semi open sets types as supra i - open set, supra regular open, regular open, regular closed, clopen, supra semi regular open set and supra clopen . By using this set , We introduce and define the notion of supra totally iR -continuous , supra iR -irresolute map and investigate some of its properties. We prove that every supra totally iR - continuous function is supra iR -irresolute, every totally $i\alpha$ -continuous function is supra totally iR -continuous and any strongly continuous function is supra totally iR -continuous, on the other hand, we give examples to show that the convers may not be true. At last , We have proved some theorem to discuss the properties of supra iR -normal space and supra iR -regular space*

Keywords: *Supra iR -Open Set, Aupra Totally iR -continuous, Supra iR -irresolute.*

1. INTRODUCTION

In 1983, Mashhour [1] explored s -continuous and s^* -continuous maps and established supra topological spaces. Thus, where the supra topological space is offered, the supra open sets are defined. Every topological space is a supra topological space, as we have known. Each open set is therefore a supra open set. The opposite, however, is not always true. It is not necessary to consider the intersection requirement to have a supra topological space. Supra α -open sets and supra α -continuous maps were created and researched as a kind of set and map between topo-logical spaces by Deve [11] in 2008. In 2012, Mohammed and Askandar [2] proposed the notion of i -open sets, which they could combine with a variety of other generalized open set concepts. In 2016, Jardo, presented the notions of supra i -open sets and discussed the notions of supra i -continuous maps, supra i -open maps, and supra i -closed maps in topological spaces[7] .The concept of supra totally π gr- continuous function in supra topological spaces



was presented by C. Janaki and V. Jeyanthi [6] in 2015. New Types of Perfectly Supra Continuous Functions was presented by Humadi and Ali in [10]. Shaheen. S. A. [12] introduced the concept of iR -open sets. Supra iR -open sets are now presented. Supra iR -open sets are characterized and properties are obtained. As applications some new classes of functions namely Supra totally iR -continuous and supra iR -irresolute map are presented. Moreover, we investigate the relationship between these functions and other related classes of functions are also developed. last, For supra iR -open sets, we provide new weak separation axioms, and we show that supra totally iR -continuous function is related to supra iR -separation axioms.

2. RESEARCH METHOD

During this paper (P, μ) (simply P) denote supra topo-logical spaces on which, unless otherwise stated, no separation axioms are imposed. Let P be a space and $E \subseteq P$, the closure of E and the interior of E will be denoted by $Cl(E)$ and $Int(E)$ respectively. The supra closure of E and supra interior of E will be denoted by $cl^\mu(E)$ and $int^\mu(E)$ respectively. $P-E$ is the complement of E .

2.1 Definition [5]

Supra topology on P refers to a subfamily of P . if :

- 1) $P, \emptyset \in \mu$.
- 2) If $E_i \in \mu$ for every one $i \in J$, then $\cup E_i \in \mu$.

A supra topo-logical space refers to the pair (E, μ) . The elements of in (E, μ) are referred to as supra open sets. while the complement of supra open set is known as supra closed set.

2.2 Definition [5]

(1) The symbol $cl^\mu(E)$ represents a set's supra closure and is defined as

$$cl^\mu(E) = \cap \{F : F \text{ is supra closed and } E \subseteq F\}.$$

(2) The symbol $int^\mu(E)$ represents a set's supra interior and is defined as

$$int^\mu(E) = \cap \{F : F \text{ is supra open and } F \subseteq E\}.$$

2.3 Definition [1]

Let (P, τ) denote a topo-logical space and μ a supra topology on P . We consider μ a supra topo-logy associated with if $\tau \subset \mu$.

2.4 Definition

The following is a subset E of a topo-logical space P :

1. regular open sets [8] if $E = Int(cl(E))$, and regular closed if $E = cl(int(E))$
2. i -open set [2][4] if $E \subseteq cl(E \cap U)$, where $\exists U \in \tau$ and $U \neq P, \emptyset$.
3. α -open set, denoted by α -OS, if $E \subseteq Int(cl(Int(E))$ [3]
4. $i\alpha$ -open set [3] if $E \subseteq Cl(E \cap U)$, where $\exists U \in \alpha O(P)$ and $U \neq \emptyset, P$.
5. If E is both open and closed, clopen is set.
6. iR -open set [12][13] if $E \subseteq Cl(E \cap U)$, where $U \in RO(E)$ and $U \neq \emptyset, P$.
7. supra regular open set [8] if $E = int^\mu(cl^\mu(E))$.
8. supra semi regular open set [9] if there is a supra regular open set U^μ such that



$$U^\mu \subseteq E \subseteq cl^\mu(U^\mu).$$

9. supra i -open set [7] if $E \subseteq cl^\mu(E \cap U^\mu)$ where $\exists U^\mu \in \mu$ and $U^\mu \neq P, \emptyset$.

Close sets are the complement of the above open sets. The most important family of all open (or. supra regular open, supra semi regular open, supra i -open, $i\alpha$ -open, supra clopen, supra iR -open, regular closed) sets of supra topo-logical space are denoted by τ (resp. $R^\mu O(P)$, $SR^\mu O(P)$, $i^\mu O(P)$, $i\alpha O(P)$, $CO^\mu(P)$, $iR^\mu O(P)$, $RC(P)$).

The definitions that follow will be helpful in the sequel.

2.5 Definition

Allow P and K be a topo-logical space, a map $M: P \rightarrow K$ is said to be :

1. strongly continuous [14] if $M^{-1}(U) \in CO(P), \forall U \in K$.
2. $i\alpha$ -totally continuous [3] if $M^{-1}(U) \in CO(P), \forall U \in i\alpha O(K)$.
3. i -irresolute [3] if $M^{-1}(U) \in iO(P) \forall U \in iO(K)$.

We recall the following definitions and results.

2.6 Lemma [12][13]

Every regular open set and regular closed set and clopen set is an iR -open set.

2.7 Lemma [12][13]

In a topo-logical space (P, τ) , every iR -open set is an $i\alpha$ -open set.

2.8 Lemma [9]

In a topo-logical space, an open set is any regular open set.

2.9 Lemma [8]

Any supra regular open is supra open.

3. RESULTS AND DISCUSSION

We now offer a different kind of supra open sets known as supra iR -open sets and examine how they relate to other supra open sets such as supra open, supra regular open, supra semi regular open, supra i -open, and supra clopen.

3.1 Definition

Consider the topo-logical space (P, μ) . If there exist a supra regular open set $U^\mu \neq P, \emptyset$ such that $E \subseteq cl^\mu(E \cap U^\mu)$, the set E is called supra iR -open set. Supra iR -closed set is the complement of supra iR -open set. $iR^\mu O(P)$ stands for the universal family supra iR -open sets of topo-logical space.

3.2 Example

Assume that (P, μ) is a supra topo-logical space with $P = \{b, a, c\}$, $\mu = \{P, \{a\}, \{b, a\}, \{c, b\}, \emptyset\}$. $iR^\mu O(P) = \{P, \{b\}, \{a\}, \{c\}, \{b, c\}, \emptyset\}$.



3.3 Proposition

Every regular open set and regular closed set and clopen set is supra iR -open set.

Proof: Due to lemma 2.6 and the fact that every iR -open set is a supra iR -open set,

3.4 Theorem

A supra iR -open set is a supra semi regular open set.

Proof : Allow any supra semi regular open set (P, μ) to be E . Then, according to the explanation given of supra semi regular open set, There is a regular open set U^μ such that

$$U^\mu \subseteq E \subseteq cl^\mu(U^\mu) \dots (1), \text{ Since } U^\mu \subseteq E \text{ then } E \cap U^\mu = U^\mu \dots (2)$$

From(1) and (2) we obtain $E \subseteq cl^\mu(A \cap U^\mu)$. Thus E is an iR -open set ■

3.5 Proposition

Each and every supra iR -open set also exist as a supra i -open set.

Proof : Allow any E be any supra iR -open set in (P, μ) . Then according definition of supra iR -open set, there exist a supra regular open set $U^\mu \neq E, \emptyset$ like that $E \subseteq cl^\mu(E \cap U^\mu)$. as a result that every supra regular open is a supra open set by lemma 2.9, Then $U^\mu \in \mu$. We obtain $E \subseteq cl^\mu(E \cap U^\mu)$ where $\exists U^\mu \in \mu$ and $U^\mu \neq P, \emptyset$. Thus E is supra i -open set ■

3.6 Proposition

There is a supra iR -open set for each supra regular open set.

Proof: Let E be any supra regular open set in (P, μ) . We prove that E is supra iR -open set if there exist $U^\mu \in R^\mu O(E)$ such that $E \subseteq cl^\mu(E \cap U^\mu)$. Since E is supra regular open set by definition of supra regular open set $E = int^\mu(cl^\mu(A))$. Put $E = U^\mu$, we get $E \subseteq cl^\mu(E \cap E) = cl^\mu(E)$. This mean A is supra iR -open set ■

The following examples shows that the convers of proposition 3.4, proposition 3.5, proposition 3.6, in general is not true.

3.7 Example

$P = K = \{a, c, b\}, \tau = \{P, \{a\}, \{b, c\}, \emptyset\}, CO(P) = \{\emptyset, \{b, c\}, \{a\}, P\}, RO(P) = \{P, \{a\}, \{b, c\}, \emptyset\} = RC(P), \mu = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, P\}, R^\mu O(P) = \{\emptyset, \{a\}, \{c, b\}, P\}, SR^\mu O(P) = \{\emptyset, \{a\}, \{c, b\}, P\}. iR^\mu O(P) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}, P\}, i^\mu O(P) = \{\emptyset, \{a\}, \{c\}, \{b\}, \{a, b\}, \{b, c\}, P\}.$

3.8 Remark

Every supra clopen is supra regular open set according to the definition of this term, and hence by proposition 3.6 is supra iR -open set.

The intersection of supra iR -open sets is not required to be supra iR -open set as showed in the example below:

3.9 Example

$P = \{a, c, b\}, \mu = \{\emptyset, \{b\}, \{a\}, \{b, a\}, P\}, iR^\mu O(P) = \{\emptyset, \{a\}, \{b\}, \{a, c\}, \{b, c\}, P\}, \text{ then } \{a, c\} \cap \{b, c\} = \{c\} \notin iR^\mu O(P).$

- As shown in Example 3.9, A supra iR -open set not necessarily their union of other supra iR -open sets. $\{a\} \cup \{b\} = \{a, b\} \notin iR^\mu O(P)$

- The supra iR -open sets are independent of supra open sets as shown in the two examples : In example 3.9 we note that $\{a,b\}$ is supra open set but it isn't supra iR -open set and in example 3.2, the set $\{b\}$ is supra iR -open set isn't supra open set.
- The following implications can be seen as a result of the foregoing discussion and known results :

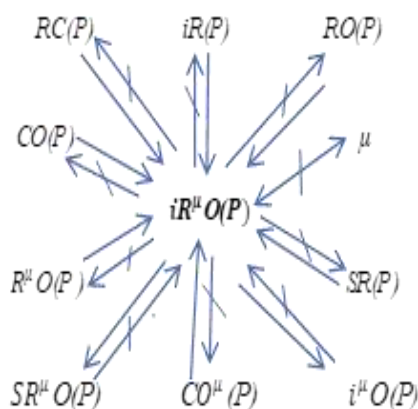


Figure1. The relationships of supra iR -open sets with other classes mentioned above

Mapping that are Supra Totally iR -Continuous and Supra iR -Irresolute

This section includes the notion of supra iR -totally cont. and supra iR -irresolute map

3.10 Definition

Consider P and K be a topo-logical space, a map $M : P \rightarrow K$ is stated to be :

- supra totally (perfectly) iR -continuous if $M^{-1}(U) \in CO^\mu(P) \forall U \in iRO(K)$.
- supra iR -irresolute, if $M^{-1}(U) \in iR^\mu O(P) \forall U \in iRO(K)$.

3.11 Proposition

Every supra totally iR - continuous function is supra iR -irresolute.

Proof : Let V be any iR -open set in K and $M : P \rightarrow K$ be supra totally iR -continuous function. M is supra totally iR -continuous function, hence, $M^{-1}(V) \in CO^\mu(P)$. Because, every supra clopen set is supra regular open set, so V is supra regular open set then by proposition 3.6 $M^{-1}(V)$ supra iR -open set in P . So M is supra iR -irresolute. The following example explains how the opposite of the aforementioned need not be true.

3.12 Example

$P = \{a,b,c\}$, $\mu = \{ \emptyset, \{c,b\}, \{a\}, P \}$, $CO^\mu(P) = \{ \emptyset, \{b,c\}, \{a\}, P \}$, $\sigma = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, K \}$, $iRO(K) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{b,c\}, K \}$, $iR^\mu O(P) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{b,c\}, P \}$ The identity mapping $M : P \rightarrow K$ is clearly supra iR -irresolute but M is not supra totally iR -continuous.

3.13 Proposition

Every supra totally iR - continuous function is supra i -irresolute



Proof: The same proof because every supra iR -open set is supra i -open set. The following example demonstrates how the opposite of the previous requirement must not be true.

3.14 Example

$P = K = \{a, b, c\}$, $\mu = \{ \emptyset, \{a\}, \{a, b\}, \{b, c\}, P \}$, $\sigma = \{ \emptyset, \{a\}, \{b, c\}, K \}$

$CO^\mu(P) = \{ \emptyset, \{a\}, \{b, c\}, \{c\}, P \}$, $iO(P) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, P \}$

$iO(K) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}, K \}$, $iRO(K) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}, K \}$.

Although f is not supra totally iR -continuous, it is obvious that the identity $f: P \rightarrow K$ is supra i -irresolute.

3.15 Theorem

Also supra totally iR -continuous. is the composition of two such maps.

Proof: let $f: P \rightarrow K$ and $g: K \rightarrow H$ any two supra iR -totally continuous, Give Y any iR -open set in H . Because g is supra iR -totally continuous, $g^{\square 1}(Y)$ is supra clopen set in K then $g^{\square 1}(Y) \in iR^\mu O(K)$ which shows $g^{\square 1}(Y)$ is supra iR -open set in K . Because f is supra totally iR -continuous then $f^{-1}g^{-1}(Y) = (gof)^{-1}(V) \in CO^\mu(P)$. Hence, $gof: P \rightarrow H$ is supra iR -totally continuous. ■

3.16 Proposition

Every totally $i\alpha$ -continuous function is supra totally iR -continuous

Proof: Allow Y be any iR -open subset in K and let $f: P \rightarrow K$ be totally $i\alpha$ -continuous function then, by lemma 2.7 Y is $i\alpha$ -open set in K . Because f is totally $i\alpha$ -continuous function, $f^1(Y)$ is clopen set in P by definition 2.5(2). Hence $f^1(Y) \in CO^\mu(P)$ set in P . So f is supra totally iR -continuous ■, As shown by the example below it is not necessary for the opposite of the above to be true.

3.17 Example

$P=K=\{c, b, a\}$, $\mu = \{ \emptyset, \{b\}, \{a\}, \{b, a\}, \{c, b\}, \{c, a\}, P \}$, $CO^\mu(P) = \{ \emptyset, \{c, b\}, \{c, a\}, \{b\}, \{a\}, \{c\}, P \}$,

$\tau = \{ \emptyset, \{a\}, \{b, c\}, P \} = CO(P)$, $\sigma = \{ \emptyset, \{b, a\}, \{a\}, \{b\}, K \}$, $i\alpha O(K) = \{ \emptyset, \{b\}, \{a\}, \{a, b\}, \{c, a\}, \{b, c\}, K \}$, $iRO(K) = \{ \emptyset, \{a\}, \{b\}, \{c, a\}, \{c, b\}, K \}$. The identity mapping $f: P \rightarrow K$ is clearly supra iR -totally cont. but f is also not totally $i\alpha$ -cont.

3.18 Proposition

Any strongly continuous function is supra totally iR -continuous.

Proof: Let $M: P \rightarrow K$ any strongly continuous function and give Y any one iR -open subset in K . Because M is strongly continuous function, $M^{\square 1}(Y)$ is clopen set in P . Hence $M^{\square 1}(Y) \in CO^\mu(P)$ set in P . So f is supra totally iR -continuous ■

3.19 Theorem

A bijective map $M: P \rightarrow K$ is supra totally iR -continuous if and only if each supra iR -closed subset of K has a $cl^\mu open^\mu$ inverse image in P .

Proof: Let C be any supra iR -closed subset in K . Then $K-C$ is supra iR -open in K . Because M is supra totally iR -continuous, $f^{\square 1}(K-C) = f^{\square 1}(K) - f^{\square 1}(C) = P - f^{\square 1}(C) \in CO^\mu(P)$ Hence $f^{\square 1}(C) \in CO^\mu(P)$. Conversely, let V be any supra iR -open subset in K . By



assumption $f^{-1}(K-V) \in CO^\mu(P)$. This denotes $P - f^{-1}(V) \in CO^\mu(P)$, $f^{-1}(V) \in CO^\mu(P)$. So f is supra totally iR -continuous. ■

3.20 Theorem

If $f: P \rightarrow K$ is supra totally iR -continuous and $g: K \rightarrow H$ is supra iR -irresolute mapping then $g \circ f: P \rightarrow H$ is iR - supra totally iR -continuous.

Proof: Assume that V is an iR -open set to H , because $g: K \rightarrow H$ is supra iR -irresolute mapping, $g^{-1}(V)$ is supra iR -open to K , Because f is supra iR -totally continuous, $f^{-1}g^{-1}(V) \in CO^\mu(P)$. Hence $g \circ f: P \rightarrow H$ is supra totally iR -continuous.

3.21 Theorem

If $f: P \rightarrow K$ is supra iR -irresolute and $g: K \rightarrow H$ is supra totally iR -continuous then $g \circ f: P \rightarrow H$ is supra iR -irresolute.

Proof: Let A any iR -open set in H , because g is supra totally iR -continuous function, $g^{-1}(A) \in CO^\mu(K)$. Then $g^{-1}(A)$ is supra iR -open in K . As f is supra iR -irresolute. After that $f^{-1}g^{-1}(A) = (g \circ f)^{-1}(A)$ is supra iR -open in P . So $g \circ f$ is supra iR -irresolute ■

New Separation Axioms

We present some new weak supra separation axioms using supra iR -open set in this part.

3.22 Definition

(1) supra iR -regular if disjoint supra iR -open sets. can be separated for any closed set C not containing a point.

(2) If there are two disjoint supra iR -open sets G, W for each disjoint closed set C_1 and C_2 of P , such that $C_1 \subset G$ and $C_2 \subset W$ then supra iR -normal.

3.23 Theorem

Let $M: P \rightarrow K$ is supra totally iR -continuous and supra iR -closed injection function, if K is supra iR -regular, then P is supra iR -regular.

Proof: Take C be a supra closed set that does not contain e . We have $M(C)$ is iR -closed set in K not containing $M(e)$ since M is iR -closed. Since, K is supra iR -regular, there is disjoint supra iR -open set D, F such that $M(e) \in D$ and $M(C) \subset F$, which imply $e \in M^{-1}(D)$ and $C \subset M^{-1}(F)$. As M is supra totally iR -continuous, $M^{-1}(D)$ and $M^{-1}(F)$ are supra clopen set in P . Then $M^{-1}(D)$ and $M^{-1}(F)$ are supra iR -open subset in P (remark 3.8). Also, since M is an injective, there are $M^{-1}(D) \cap M^{-1}(F) = M^{-1}(D \cap F) = M^{-1}(\emptyset) = \emptyset$. disjoint supra iR -open sets $M^{-1}(D)$ and $M^{-1}(F)$ separate a point e and a closed set C that does not include e . As a result, P is an iR -regular.

3.24 Theorem

If $M: P \rightarrow K$ is supra totally iR -continuous and supra closed injection while K is supra iR -normal. Then P is supra iR -normal.

Proof: Take C_1 and C_2 be disjoint supra closed subset of P . Because M is supra closed, $M(C_1)$ and $M(C_2)$ are disjoint supra closed subset in K , and because K is supra iR -normal, $M(C_1)$ and $M(C_2)$ are separated by disjoint supra iR -open subset G and W of K such that $C_1 \subset$



$M^{\square 1}(G)$ and $C_2 \subset M^{\square 1}(W)$. Since M is supra totally iR -continuous $M^{\square 1}(G)$ and $M^{\square 1}(W)$ are supra clopen set in P . Which imply $M^{\square 1}(A)$ and $M^{\square 1}(B)$ are supra iR -open sets. Also, we have $f^{\square 1}(G) \cap f^{\square 1}(W) = f^{\square 1}(G \cap W) = M^{\square 1}(\emptyset) = \emptyset$. Thus, In P , each pair of non-empty supra closed sets can be separated by iR -open sets. Therefore, P is supra iR -normal ■

4. CONCLUSIONS

We were able to define a new class of open set type- i and compare them with other semi-open sets using this definition. And we found that there is a close correlation between these classes and supra regular open, supra semi regular open, supra i -open, $i\alpha$ -open, supra clopen and regular closed sets. The aforementioned correlation is the same for closed sets.

Acknowledgment

Out of loyalty, appreciation and recognition, I would like to thank those sincere people who spared no effort in helping us in the field of scientific research.

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