Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

Optimization of Problems with Multi-Objective Functions and their Applications in Engineering

Ras Bihari Soni[1], Dr. Dharamender Singh[2*], Dr. K. C. Sharma[3]

[1]Department of Mathematics, Govt. Birla P.G. College, Bhawani Mandi, Jhalawar, Rajasthan affiliated by University of Kota, Kota, Rajasthan, India.
[2,3]Department of Mathematics, Maharani Shri Jaya Govt. College, Bharatpur, Rajasthan affiliated by Maharaja Surajmal Brij University, Bharatpur, Rajasthan, India.*

Email: [1]rbsoni68@gmail.com, [3]ks.pragya@gmail.com
Corresponding Email: [2]dharamender.2015dr7@am.ism.ac.in*

Received: 28 November 2023 **Accepted:** 14 February 2024 **Published:** 01 April 2024

***Abstract:** Many real-world optimization issues typically have multiple competing goals. There is generally no solution in those multi-objective optimization problems that optimizes all objective functions at the same time. Rather, "efficient" in terms of all objective function's solutions known as Pareto optimum solutions are presented. We typically have a large number of Pareto-optimal options. As a result, we must choose a final solution from among Pareto optimal solutions while considering the objective function balance; this process is known as "trade-off analysis." It is not hyperbole to state that trade-off analysis is the most crucial task in multi-objective optimization. As a result, the methodology's ease of use and comprehension for trade-off analysis should be highlighted.*

The set of Pareto optimal solutions in the objective function space, or Pareto frontier, can be represented somewhat simply in circumstances when there are two or three objective functions. We can fully understand the trade-off relationship between objectives when we see Pareto boundaries. Thus, in scenarios with two or three objectives, it would be the most appropriate technique to represent Pareto borders. Reading the trade-off relationship between objectives with three dimensions, however, may be challenging. Nonetheless, Pareto frontier cannot be represented in situations where there are more than three objectives. In this case, interactive techniques can assist us in performing a local trade-off analysis that reveals a "certain" Pareto optimal option. Many techniques have been devised, with variations in which the Pareto optimal solution is displayed. The main concerns of such multi-objective optimization techniques, especially as they relate to engineering design problems, are covered in this study.

***Keywords:** Multi-Objective Optimization, Pareto Frontier, Evolutionary Algorithms.*

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

1. INTRODUCTION

The following is how multi-objective programming challenges are expressed-

Minimization $f(x) \equiv \{f_1(x), f_2(x) \ldots\ldots\ldots f_r(x)\}$ Over $x \in X$

The constraint set X can be written as –
$g_j(x) \leq 0 \quad j = 1,2,3 \ldots\ldots\ldots m$
The constraint set X is also a subset of R^n.
For the Multi-Objective Optimization, the Pareto solutions can be defined as following-

When $\hat{x}$ is the only superior answer $x \in X$, it is said to be Pareto optimum; that is, if

$$f(x) \not\leq f(\hat{x}) \text{ for each } x \neq \hat{x} \in X$$

There might be a lot of Pareto solutions in general. They come to a final conclusion by considering the overall balance of all the factors. This is a decision maker's (abbreviated as DM) value judgment dilemma. A complete balancing of criteria is commonly referred to as a trade-off. It should be mentioned that there are a lot of criteria—let's say more than 100—in certain real-world issues, such camera lens design and cable stayed bridge erection management. As a result, it's critical to create efficient techniques that enable DM to trade-off with ease, especially in situations with a large number of criteria. Interactive multi-objective programming asks the DM questions about his or her values while working together to find a solution. Accordingly, a number of noteworthy techniques were created in the 1980s: Of these, the aspiration level approach is now acknowledged as being particularly effective in practice because:
(i) aspiration levels accurately reflect DM's wish;
(ii) it does not require DM's judgment to be consistent;
(iii) aspiration levels function as a probe more effectively than weight for objective functions.

We shall talk about the weighting method's challenges in the following, as it is a popular approach to traditional goal programming.

2. RELATED WORK

2.1 What Makes the Weighting Method not so Effective?

The value assessment of DM is the basis for the ultimate decision in multi-objective programming issues. Therefore, it matters how we extract DM's value assessment. The vector objective function is scalarized in many real-world scenarios so that the DM value judgment can be included.
The linearly weighted sum is the scalarization method that is most widely known:

$$\sum_{i=1}^{r} w_i f_i(x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

DM's value judgment is indicated by the weight. This kind of scalarization has a significant disadvantage, while being widely utilized in many real-world scenarios. In particular, because

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

of the "duality gap," it cannot give a solution among sunken portions of the Pareto surface in nonconvex circumstances. Even in convex situations, like linear scenarios, we can only obtain a vertex of the Pareto surface if we apply the well-known simplex approach in order to find a point in the middle of a line segment between two vertices. This suggests that the linearly weighted sum may not always be able to give DM the best solution, depending on how complex the situation is.

In classical goal programming (Charnes-Cooper, 1961), the decision maker's (DM) preference is expressed as a metric function from the goal f ∗. For example, the following is well-known:

$$(\sum_{i=1}^{r} w_i|f_i(x) - f_i^*|^p)^{1/p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

The weight w_i, the value of p, and the goal value f_i^* all represent DM's desire. By minimizing the function (2.2), a Pareto solution among a sunken portion of the Pareto surface can be produced if the value of p is selected suitably. Nevertheless, figuring out what values are appropriate in advance is typically challenging. Furthermore, even though the objective is underestimated, the solution minimizing (2.2) cannot be superior to the aim $\mathbf{f}^*$.

One of the biggest problems with goal programming is that individuals often think that changing the weight would lead to a positive outcome, which is not the case. As the following example illustrates, it should be noted that there is no positive link between the weight w_i and the value $f_i(\hat{x})$, which corresponds to the final solution $\hat{x}$.

Example 2.1 Given the equations $y_1 = f_1(x), y_2 = f_2(x)$ and $y_3 = f_3(x)$, the feasible region in the objective space can be represented as follows:

$$\left\{(y_1, y_2, y_3) \mid (y_1 - 1))^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \leq 1\right\}.$$

Assume that $(y_1^*, y_2^*, y_3^*) = (0,0,0)$ is the objective. With p=1 and w_1= 1,w_2=1 and w_3 =1, the solution that minimizes the metric function (ii) is $(y_1, y_2, y_3) = (1 - 1/\sqrt{3}, 1 - 1/\sqrt{3}, 1 - 1/\sqrt{3})$. Let's say that DM want to significantly reduce the values of f_1 and f_2, changing the weight to $w_1' = 10, w_2' = 2, w_3' = 1$. The new weight's corresponding solution is $(1 - 10/\sqrt{105}, 1 - 2/\sqrt{105}, 1 - 1/\sqrt{105})$. Observe that although though DM intends to improve it and has raised the weight of f_2 up to twice, the value of f_2 is worse than it was before. Someone can believe that the lack of weight normalization is the cause of this. We therefore use $w_1 + w_2 + w_3 = 1$ to standardize the weight. The refreshed weight with the same normalization is $w_1' = 10/13, w_2' = 2/13, w_3' = 1/13$. The original weight, normalized in this manner, is $w_1 = w_2 = w_3 = 1/3$.

It is evident that w_2' is smaller than w_2. Next, raise w_2 to a normalized weight greater than 1/3. Set the unnormalized weights $w_1 = 10, w_2 = 7$ and $w_3 = 1$ in order to achieve this. We have a solution $(1 - 10/\sqrt{150}, 1 - 7/\sqrt{150}, 1 - 1/\sqrt{150})$ with this new weight. Even if the acquired solution is better than the previous one, the normalized weight $w_2'' = 7/18$ is still more than the original one (=1/3).

It is normally quite difficult to change the weight in order to get the desired result, as the example above makes clear. As a result, it would appear preferable to use DM's ambition level

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

as the probe rather than weight. The limitations of traditional goal programming can be addressed by developing interactive multi-objective programming techniques based on aspiration levels. One of them, the satisficing trade-off method established by the author (Nakayama 1984), will be covered in the part that follows.

2.2 Trade-off Satisficing Method

The aspiration level at the k-th iteration $\bar{f}^k$ is altered in the aspiration level method in the following ways:

$$\bar{f}^{k+1} = T \circ P(\bar{f}^k) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

The Pareto solution that is, in some way, closest to the specified aspiration level $\bar{f}^k$ is chosen in this case by operator P. If DM does not agree to a compromise with the proposed solution $P(\bar{f}^k)$, the trade-off operator T modifies the k-th aspiration level $\bar{f}^k$. Naturally, because $P(\bar{f}^k)$ is a Pareto solution, there isn't a workable solution that improves on $P(\bar{f}^k)$ for every criterion. As a result, DM must make trade-offs between criteria in order to improve on some of them. $T \circ P(\bar{f}^k)$ is chosen as the new aspiration level based on this trade-off. This approach is carried out repeatedly until DM finds a workable answer. This concept is applied in the satisficing trade-off approach (Nakayama 1984) and DIDASS (Grauer et al. 1984). The satisficing trade-off technique offers a device based on the sensitivity analysis, whereas DIDASS leaves the trade-off to the heuristics of DM.

2.3 On the Operation P

Some auxiliary scalar optimization performs the operation which yields a Pareto solution $P(\bar{f}^k)$ nearest to $\bar{f}^k$. Sawaragi-Nakayama-Tanino (1985) demonstrated that the Tchebyshev norm type is the only scalarization technique that yields a Pareto solution for any problem, regardless of its structure. On the other hand, the Tchebyshev norm type scalarization function produces both a Pareto and a weak Pareto solution. Weak Pareto solutions are not always "efficient" as a solution in decision making because there is a chance that another solution exists that improves a criterion while fixing others. The augmented Tchebyshev type scalarization function of the following kind can be applied to eliminate weak Pareto solutions:

$$\max_{1\le i\le r} w_i\big(f_i(x) - \bar{f}_i\big) + \alpha \textstyle\sum_{i=1}^{r} w_i f_i(x) \ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

where α is often set to a minimally significant positive value, such as 10^{-6}.

Theorem 2.1 (Tanino- Nakayama) Minimizing (II) is a correctly Pareto optimum solution to (MOP) for arbitrary $w \geq 0$ and $\alpha > 0, \hat{x} \in X$. On the other hand, if $\hat{x}$ minimizes (II) over X, then there exist $w > 0, \alpha > 0$, and $\bar{f}$ such that $\hat{x}$ is a correctly Pareto optimum solution to (MOP).

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

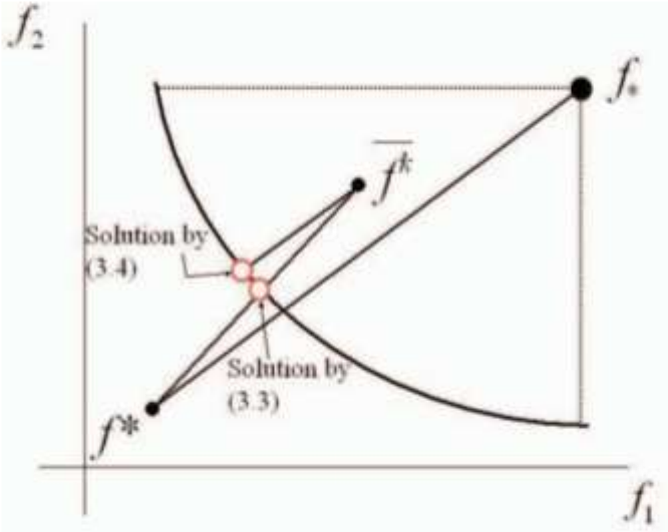


Fig. 1 Pareto Solutions' Differences by (III) and (IV)

Typically, the weight w_i is provided as follows: An ideal value, denoted as f_i^*, is often provided so that $f_i^* < Min\{f_i(x) \mid x \in X\}$. In this case, we set

$$w_i^k = \frac{1}{\bar{f}_i^k - f_i^*} \qquad \text{(III)}$$

If it is preferred that the weight remain the same despite the aspiration level changing, it can be provided by

$$w_i^{k'} = \frac{1}{f_{*i} - f_i^*} \qquad \text{(IV)}$$

In this case, the nadir value, f_{*i}, is typically can be written as-

$$f_{*i} = \max_{1 \le j \le r} f_i(x_j^*) \qquad \text{(V)}$$

While

$$x_j^* = arg \min_{x \in X} f_j(x) \qquad \text{(VI)}$$

Since (II) is not smooth, the minimizing of (II) using (III) or (IV) is typically accomplished by resolving the analogous optimization problem as follows:

Minimize

$$z + \alpha \sum_{i=1}^{r} w_i f_i(x)$$

subject to

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

$$w_i^k\left(f_i(x) - \bar{f}_i^k\right) \le z \quad x \in X. \quad \text{...(VII)}$$

Note- Fig. 1 shows the variation in solutions to (Q) for two types of weights (III) and (IV). It is possible to substitute f_i^* for $\bar{f}_i^k$ in the constraint (VII) of the auxiliary min-max problem (Q) with the weight by (III) without affecting the solution. Since we have

$$\frac{f_i(x)-f_i^*}{\bar{f}_i^k-f_i^*} = \frac{f_i(x)-\bar{f}_i^k}{\bar{f}_i^k-f_i^*} + 1 \text{.....................(VIII)}$$

2.4 Regarding Operation T

If the DM is not happy with the answer for $P(\bar{\boldsymbol{f}}^k)$, then he or she is asked to respond with $\bar{\boldsymbol{f}}^{k+1}$, which is their new aspiration level. Assume that x^k represents the Pareto solution that was achieved through projection $P(\bar{\boldsymbol{f}}^k)$. Then, organize the objective functions into three categories:

There are three categories of criteria: (i) those that need further improvement; (ii) those that can be loosened; and (iii) those that are fine as they are.

For each class, let I_I^k, I_R^k *and* I_A^k and stand for the index set, respectively. For all i in I_I^k, $\bar{f}_i^{k+1} < f_i(x^k)$ is evident. Typically, $\bar{f}_i^{k+1} = f_i(x^k)$is set for $i \in I_A^k$. DM must consent to raise the value of $\bar{f}_i^{k+1}$ for $i \in I_R^k$. It should be emphasized that in order to improve f_i for $i \in I_I^k$, a suitable sacrifice of f_j for $j \in I_R^k$ is required.

Example 2.2 Think about the identical issue as in Example 2.1: Given the equations $y_1 = f_1(x), y_2 = f_2(x)$ and $y_3 = f_3(x)$, the feasible region in the objective space can be represented as follows:

$$\left\{(y_1, y_2, y_3) \mid (y_1 - 1)\right)^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \le 1\Big\}.$$

Assume that $(y_1^*, y_2^*, y_3^*) = (0,0,0)$ is the ideal point and $(y_{*1}, y_{*2}, y_{*3}) = (1,1,1)$ is the nadir point. As a result, we have $w_1 = w_2 = w_3 = 1.0$. using (VI). Assume that $(\bar{y}_1^1, \bar{y}_2^1, \bar{y}_3^1) = (0.4,0.4,0.4)$ is the initial aspiration level. Hence, $(y_1^1, y_2^1, y_3^1) = (0.423,0.423,0.423)$ is the answer to (Q). Let's say that DM wants to change the aspiration level to $\bar{y}_1^2 = 0.35$ and $\bar{y}_2^2 = 0.4$ by significantly lowering the values of f_1 and f_2. It is not possible to enhance all of the criteria because the current solution, $(y_1^1, y_2^1, y_3^1) = (0.423,0.423,0.423)$ is already Pareto optimum.

Consequently, let's say that DM consents to easing f_3 and adopting $\bar{y}_3^2 = 0.5$ as the new aspiration goal. The answer to (Q) with this new ambition level is $(y_1^2, y_2^2, y_3^2) = (0.366,0.416,0.516)$. It should be highlighted that the acquired solution is more improved than the previous one, even though it slightly falls short of the aspiration level of f_1 and f_2. The reason f_1 and f_2's improvement falls short of DM's desired level is because f_3's relaxation is insufficient to offset f_1 and f_2's improvement. By conducting a purposeful trade-off analysis, DM may quickly find a satisfying solution using the satisficing trade-off method.

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

2.5 The Swapping of Constraints and Objectives

To formulate the auxiliary scalarized optimization problem (Q), substitute $\beta_i z$ for the right-hand side of equation (VII).

$$w_i\left(f_i(x) - \bar{f_i}\right) \leq \beta_i z \text{ ....(IX)}$$

It is clear that the function f_i is regarded as an objective function if $\beta_i = 1$, meaning that while the level of f_i should ideally be higher, the ambition level $\bar{f_i}$ need not always be reached. Conversely, f_i is regarded as a constraint function for which the ambition level $\bar{f_i}$ should be assured if $\beta_i = 0$. Almost never do we regard the function of the objective and the restriction to be fixed from the outset in real problems; instead, we typically desire to swap them around based on the circumstances. This is simply accomplished using formula (IX) (Korhonen 1987). Furthermore, f_i may have a role in the midst of the goal and constraint, which is not a complete objective nor a complete constraint, if the value of β_i is set between 0 and 1 (Kamenoi et al. 1992). This works wonders in a lot of real-world scenarios as well.

Applications

Multi-objective interactive programming techniques have been used to solve many real-world issues. Eschenauer et al. provide some excellent examples of engineering applications (1990). Additionally, the satisficing trade-off approach has been used to solve a number of actual issues:

1. Mixing

- livestock feed formulation.
- materials of Plastic.
- cement manufacturing.
- portfolio formation.

2. Designing

- camera lens.
- cable-stayed bridge erection management.

3. Organizing

- long-term atomic power plant planning.
- In the steel industry, string selection is scheduled.

A basic explanation of how the satisficing trade-off strategy is applied to cable-stayed bridge erection management is provided below. The following standards are taken into account for accuracy control when building a cable-stayed bridge:

- residual camber error at every node,
- how many cables need to be changed,
- residual inaccuracy for every cable tension,
- the shim adjustment amount for every cable.

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

The residual error in each cable tension and that in each camber are both linear functions of the amount of shim adjustment because the change in cable rigidity is negligible enough to be overlooked in relation to shim adjustment. x_{ik} is the change in tension of the i-th cable as a result of a unit change in the k-th cable length, and ΔT_i $(i = 1, \dots, n)$ is defined as the difference between the planned and measured tension values with n as the number of cables in use. Following the shim modification, the residual cable tension error is given by-

$$p_i = \left| \Delta T_i - \sum_{k=1}^{n} x_{ik} \Delta l_k \right| \; (i = 1, \dots, n)$$

Let m be the number of nodes, y_{jk} the camber change at the j-th node brought on by the k-th cable length changing by a unit, and Δz_j $(j = 1, \dots, m)$ the difference between the specified and measured camber values. Next, assuming the shim adjustments of $\Delta l_1, \dots, \Delta l_n$, the residual error in the camber is given by

$$q_j = \left| \Delta Z_j - \sum_{k=1}^{n} y_{jk} \Delta l_k \right| \; (j = 1, \dots, m)$$

Furthermore, the shim adjustment amount can be regarded as an objective function of

$$r_i = |\Delta l_i| \; (i = 1, \dots, n)$$

Furthermore, the following are the top and lower limits of shim adjustment built into the cable anchorage structure:

$$\Delta l_{Li} \le \Delta l_i \le \Delta l_{Ui} \; (i = 1, \dots, n)$$

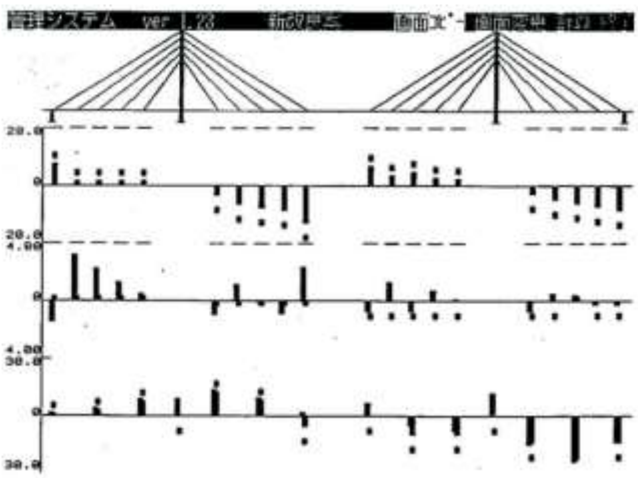


Fig. 2 System for Erection Management in Cable-stayed Bridges

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

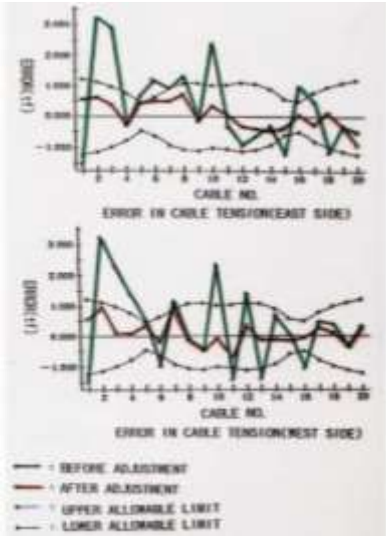


Fig. 3 Erection Management System outcome

Using the satisficing trade-off strategy, the monitor output of the cable stayed bridge erection management system is shown in Fig. 2, and the result is shown in Fig. 3. Graphs display the shim adjustment amount in addition to the residual error for each criterion. To enter the ambition level on the graph, use a mouse. Similar to this, once the auxiliary min-max problem has been resolved, a graph depicts the Pareto solution based on the aspiration level. This method is continued until the designer gets the appropriate shim-adjustment. Notable features are the visual information on criterion trade-off's ease of use and the designer's ease of operation.

The software was utilized for actual bridge construction projects, including as the 1992 construction of the Karasuo Harp Bridge in Kita-Kyusyu City and the Swan Bridge in Ube City.

2.6 Pareto Frontiers Generation

It is optimal to give Pareto frontiers when working with two or three objective functions if evaluating each one doesn't require a lengthy time. This allows DM to fully comprehend the trade-off relationship between the objectives. Since we find it more difficult to comprehend the trade-off relation for three-dimensional Pareto frontiers without rotation, it would be most effective to illustrate Pareto frontiers in scenarios where there are two objectives. The constraint transformation method, sometimes referred to as the \epsilon constrained method in some books, can be used to achieve this goal. Early multi-objective optimization research demonstrates the use of this strategy (Edgeworth [6]). Since a rough but reasonable approximation of Pareto limits can usually be obtained at 10–20 sample values of the right hand side of the objective function translated into a constraint, the method works well for our purpose, as long as each optimization doesn't take a lengthy time. However, if the auxiliary optimization problems are difficult to solve with traditional optimization tools (e.g., if the problems are severely nonlinear with multi-modal, combinatorial, nonsmoothed, and so on), then the constraint transformation approach becomes challenging to depict the Pareto frontier.

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

3. METHODOLOGY

The study of using evolutionary algorithms to determine Pareto frontiers has advanced significantly in recent years. Evolutionary algorithms have been found to perform exceptionally well, especially when it comes to multi-modal, discrete, and nonsmooth objective function optimization. While the constraint transformation approach can be used to apply evolutionary algorithms for optimization, the primary goal of evolutionary algorithm research is to directly obtain Pareto frontiers. The goal of evolution is to get individuals closer to the Pareto frontier. According to this approach, what matters are the rates at which individuals converge to the Pareto frontier and their degree of dispersion across the entire Pareto frontier. In order to do this, numerous researchers have published a variety of devices for fitness functions and evolutionary operators.

The ranking approach is one of the most used evolutionary techniques. While various techniques have been devised to assess the diversity of individuals on the Pareto frontier, the fundamental concept is in measuring each individual's distance from the frontier in relation to the number of dominant individuals. Nevertheless, the rank does not reflect the actual "distance" between each person and the Pareto frontier. The application of Data Envelopment Analysis (DEA) to produce the Pareto frontier was suggested by Arakawa et al. [1]. DEA was first created by Charnes et al. [3] to assess the effectiveness of decision-making units. Its goal is to solve a linear programming problem in order to determine the "distance" between each decision unit and the Pareto frontier. A portion of the convex hull of decision units approximates the true Pareto frontier. Multiple applications have shown that DEA gives the Pareto frontier with comparatively fewer people. This indicates that in engineering design issues, the Pareto frontier can be reached with fewer experiments (analysis). DEA cannot, however, generate a nonconvex Pareto frontier in its current form since it is predicated on the convex hull of decision units. In order to make DEA applicable to non-convex instances, Yun et al. extended DEA to GDEA (Generalized Data Envelopment Analysis) , and they then used GDEA to produce the Pareto frontier . Our experiences have shown that GDEA produces a well-distributed Pareto frontier with fewer experiments (analysis). Additionally, Yun et al. attempt to apply several computational intelligence techniques, such as Support Vector Machine (SVM) , which was first created for machine learning pattern categorization. The main goal of machine learning algorithms like Support Vector Machines (SVM) is to estimate the discriminant border for classification. Specifically, the recently developed \nu-SVM is particularly useful for single-category situations. For issues in a single category, this condition allows one to estimate not just the Pareto border but also the feasible region. It has been noted that while SVM does not always offer the Pareto frontier, in certain situations it does. In this regard, more investigation ought to be done.

4. RESULT AND DISCUSSION

We will compare the strategies mentioned in this paragraph using many computer simulations.

i) Problem related to Cantilever Beam

As illustrated in Fig. 4 , consider a cantilever design problem with two design variables: diameter (d) and length (l). P is the end load that the beam must support. The cantilever design

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

problem includes two competing design goals, namely, minimizing end deflection f_2 and weight f_1, as well as two constraints:

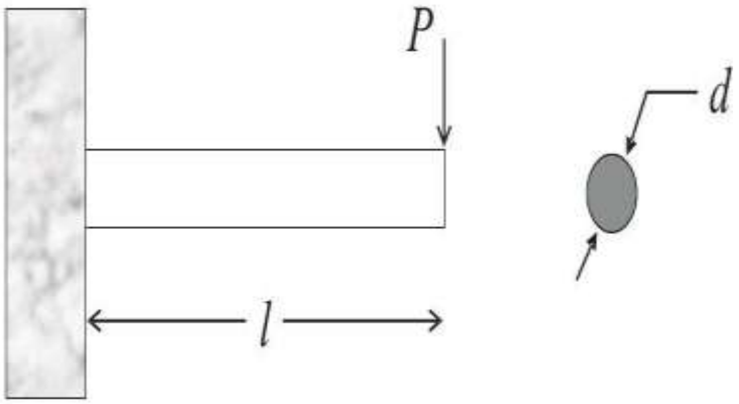
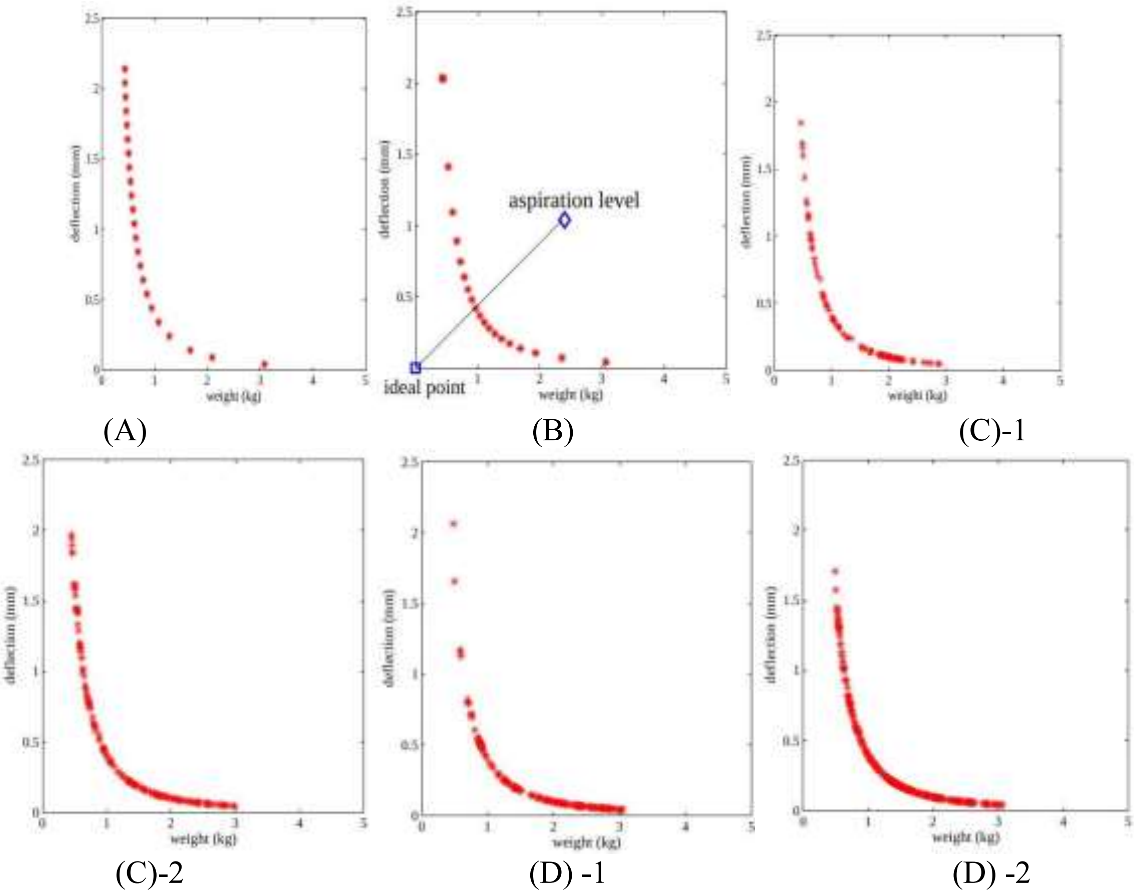
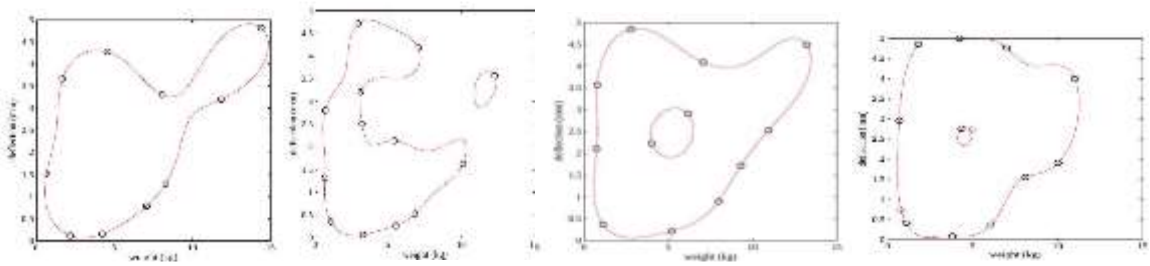


Fig. 4 A Schematic for Cantilever Beam Design

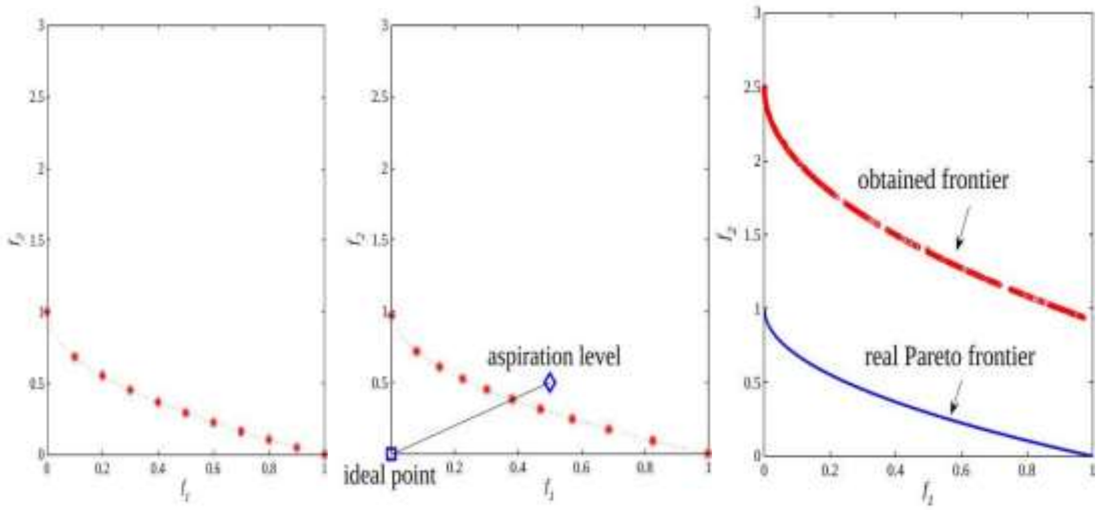


Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33



(E)-1 (E)-2 (E)-3 (E) -4

Fig. 5 Comparing the Outcomes with: (A) method of ε−constraint (B) method of satisficing trade-off (C) MOGA (D) GDEA (E) SVM



(A) (B) (C)

Fig. 6 Analyzing the Results in Relation to Problem ZDT4 by: (A) method of ε−constraint (B) method of satisficing trade-off (C) GDEA

The final deflection δ is less than a given limit t δ_{max}, and the resulting maximum stress σ_{max} is less than the permitted strength S_y. The optimization problem is now stated in the following manner:

$$\begin{aligned} &\text{minimize} \quad f_1(d,l) := \rho\frac{\pi d^2}{4} l \\ &\text{min} \quad f_2(d,l) := \delta = \frac{64Pl^3}{3E\pi d^4} \\ &\text{s. t.} \quad \sigma_{max} \leqq S_y \\ &\quad \delta \leqq \delta_{max} \\ &\quad 10 \leqq d \leqq 50, 200 \leqq l \leqq 1000 \end{aligned}$$

when the following formula is used to determine the maximum stress:

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

$$\sigma_{max} = \frac{32Pl}{\pi d^3}$$

The following is how the parameter values are used:

$$\rho = 7800\ kg/m^3, P = 1kN,\ E = 207GPa,$$
$$S_y = 300MPa,\ \delta_{max} = 5\ mm.$$

ii) ZDT4 Problem
ZDT4, as an example that is difficult for MOGA to solve, is proposed by Zitzler, Deb, and Thiele.

$$\underset{\mathbf{x}}{\text{minimize}} \quad f_1(\mathbf{x}) = x_1$$
$$f_2(\mathbf{x}) = g(\mathbf{x}) \times \left(1 - \sqrt{\frac{f_1(\mathbf{x})}{g(\mathbf{x})}}\right)$$
$$\text{subject to} \quad g(\mathbf{x}) = 1 + 10(N-1) + \sum_{i=2}^{N} \left(x_i^2 - 10\cos(4\pi x_i)\right),$$
$$x_1 \in [0,1], x_i \in [-5,5], i = 1,2,\dots, N(N = 10).$$

There are two objective functions and ten design variables. The solution to ZDT 4's Pareto optimum values is $g(\mathbf{x}) = 1$.

Table 1 and Figs. 5 and 6 display the answers to our test issues. The computation was carried out using MATLAB version 6.5.

Table. 1Comparison of the quantity of calls to functions Beam Design Problem

<table>
<tr><th>Method of ε−constraint</th><th>Method of satisficing trade-off</th><th>MOGA & GDEA</th><th>SVM</th></tr>
<tr><td rowspan="4">951
Figure 5 (A)

cf. # ε : 23 cases</td><td rowspan="4">52 per one aspiration level
Figure 5 (B)

cf. # on the average</td><td rowspan="2">1000 (10 generation× 100 data)
Figure 5 (C)-1, (D)-1</td><td>250 (5 generation× 50 data)
Figure 5 (E)-1</td></tr>
<tr><td>500 (10 generation 50 data)
Figure 5 (E)-2</td></tr>
<tr><td rowspan="2">1500 (15 generation ×100 data)
Figure 5 (C)-2, (D)-2</td><td>1000 (10 generation× 100 data)
Figure 5 (E)-3</td></tr>
<tr><td>1500 (15 generation×100 data)
Figure 5 (E)-4</td></tr>
</table>

1) ZDT4 Problem

Method of	Method of satisficing	GDEA	SVM

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

ε−constraint	trade-off		
613 Figure 6 (A) cf. # ε : 11 cases	264 per one aspiration level Figure 6 (B) cf. # on the average	25000 (250 generation ×100 data) Figure 6 (C)	—-

Using either strategy, nearly identical answers were obtained for the cantilever beam design problem. It is commonly recognized that ZDT4 is a challenging problem for evolutionary algorithms to tackle. We employed a basic GA in the internal procedure for implementing GDEA. As a result, the precise Pareto border could not be obtained using GDEA or MOGA using simple GA. Therefore, more advanced evolutionary algorithms ought to be used in this situation. SVM was unable to provide a viable solution for this problem in the number of functions calls in the same order.

Note that in situations when traditional optimization approaches can be used, classical methods like the constraint transformation method (ϵ constraint method) can produce a Pareto frontier with fewer function calls. It's unclear how the problem is handled by MATLAB's nonlinear optimization function. It requires roughly 400 function calls for each ambition level when using a separate software that was built by one of the author's coworkers and is based on SQP with numerical differentiation. Because ZDT4 has 10 variables, the number of function calls can be reduced by up to 1/10 if the derivatives of the functions are accessible.

5. CONCLUSION

Helping decision makers to a good conclusion by balancing multiple competing objectives is one of the main goals of multi-objective optimization. While it is optimal to show the entire Pareto frontier in order to fully understand the trade-off relationship between objectives, this is difficult to do when there are three or more objectives. Consequently, situations having two objectives are most suited for this strategy. Evolutionary approaches have been wonderfully developed for this goal in recent years. But those techniques typically require a large number of users, or a large number of function calls. Function evaluation is generally costly and time-consuming in engineering design challenges. because they are determined using a variety of investigations, including fluid mechanical, thermodynamic, and structural analyses, among others, and occasionally even actual sample preparation. Parallelization is a tool used in the computation of evolution to get around this problem. An additional tool involves assessing the fitness of a subset of individuals and utilizing artificial intelligence to approximate the fitness of others.

It's crucial to keep in mind that those strategies work best in situations where there are just two or three goals. In this context, the conventional constraint transformation approach (ϵ constraint method) can be implemented with fewer function calls, as demonstrated in this study. I think it's crucial to remember the fundamentals of multi-objective optimization, and there are instances when it makes sense to return to the old favorites. In an experiment with more than three objectives, the combination of an aspiration level approach and Pareto border generation was used.

Multi-objective optimization issues involve human value judgment, which is one of the most notable distinctions between them and the natural sciences. Decision-makers frequently make inconsistent value judgments at different stages of the decision-making process. Obtaining a solution that accurately reflects the decision makers' value assessment is crucial, even if it is not always consistent. Therefore, using a computer alone to find a solution would be nearly impossible; nonetheless, in many engineering design challenges, human-computer cooperation is inevitable. It is crucial to utilize the advantages of both humans and computers in this situation. In light of this, it may be feasible and efficient to combine human-side satisficing and computer-side optimization, particularly when dealing with multi-objective optimization issues in real-world scenarios.

6. REFERENCES

1. Arakawa, M., Nakayama, H., Hagiwara, I. and Yamakawa, H. (1998), Mult objective Optimization Using Adaptive Range Genetic Algorithms with Data Envelopment Analysis, A Collection of Technical Papers on 7th Symposium on Multidisciplinary Analysis and Optimization (TP98-4970), AIAA, 3, 2074–2082.
2. Charnes, A. and Cooper, W. (1961), Management Models and Industrial Applications of Linear Programming, Vol. 1, John Wiley, New York.
3. Charnes, A., Cooper, W.W. and Rhodes, E. (1978), Measuring the Efficiency of Decision-Making Units, European Journal of Operational Research, 2, 429-444.
4. Coello Coello, C.A., Van Veldhuizen, D.A. and Lamont, G.B. (2002), Evolutionary Algorithms for Solving Multi-Objective Problems, Kluwer Academic Publishers.
5. Nakayama, H. (1984), Proposal of Satisficing Trade-off Method for Multi- objective Programming, Transact. SICE, 20: 29-35 (in Japanese)
6. Nakayama, H. (1989), An Interactive Support System for Bond Trading, in A. G. Lockett and G. Islei (eds.) Improving Decision Making in Organizations, Springer, 325-333
7. Nakayama, H. (1991), Satisficing Trade-off Method for Problems with Multiple Linear Fractional Objectives and its Applications, in A. Lewandowski and V. Volkovich (eds.) Multiobjective Problems of Mahtematical Programming, Springer, 42-50.
8. Nakayama, H. (1992), Trade-off Analysis using Parametric Optimization Techniques, European J. of Operational Research, 60, 87-98
9. Nakayama, H. (1995), Aspiration Level Approach to Interactive Multi-objective Programming and its Applications, P.M. Pardalos, Y. Siskos and C. Zopounidis (eds.), Advances in Multicriteria Analysis, Kluwer Academic Publishers, 147-174.
10. Nakayama, H., Nomura, J., Sawada, K. and Nakajima, R. (1986). An Application of Satisficing Trade-off Method to a Blending Problem of Industrial Materials, in G. Fandel et al. (Eds.), Large Scale Modelling and Interactive Decision Analysis. Springer, 303-313.
11. Nakayama, H. and Sawaragi, Y. (1984). Satisficing Trade-off Method for Interactive Multiobjective Programming Methods, in
12. M. Grauer and A.P. Wierzbicki (eds.) Interactive Decision Analysis, Proceedings of an International Workshop on Interactive Decision Analysis and Interpretative Computer Intelligence, Springer: 113-122.

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 04, No. 03, April-May 2024
http://journal.hmjournals.com/index.php/JECNAM
DOI: https://doi.org/10.55529/jecnam.43.18.33

13. Nakayama, H. and Tanino, T. (1994): Multi-objective Programming-Theory and Applications, Society of Instrument and Control Engineering, (in Japanese)
14. Nakayama, H., Kaneshige, K., Takemoto, S. and Watada, Y. (1995), An Application of a Multi-objective Programming Technique to Construction Accuracy Control of Cable-stayed Bridges
15. Yun, Y.B., Nakayama, H., Tanino, T. and Arakawa, M. (2001), Generation of Efficient Frontiers in Multi-Objective Optimization Problems by Generalized Data Envelopment Analysis European Journal of Operational Research, 129 (3), 586–595.
16. Yun, Y.B., Nakayama, H. and Arakawa, M. (2004), Multiple Criteria Decision Making with Generalized DEA and an Aspiration Level Method, European Journal of Operational Research, 158, 697-706
17. Yun, Y.B., Nakayama, H., Arakawa, M. and Shiraki, W. (2004), Multi-objective Optimization Technique using Computational Intelligence, to be presented at CJK-OSM3