

Research Paper



A hybrid iterative-series method for solving kepler's equation with enhanced convergence

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ABSTRACT

Kepler's equation, a fundamental relation in celestial mechanics, poses significant numerical challenges due to its transcendental nature and the nonlinearity introduced by eccentricity. Existing iterative and series-based methods, while efficient in specific domains, often fail to deliver uniformly fast and stable convergence across the full eccentricity range. This paper introduces a hybrid solution method that dynamically combines a truncated Fourier-Bessel series for low-eccentricity scenarios with an optimized Newton-Halley iterative scheme for moderate-to-high eccentricities. A switching method in line with Laplace's limit is used for robust convergence around the threshold eccentricity. The numerical experiments demonstrate that the method proposed, in comparison to classical methods, achieves better results for convergence speed, numerical stability, and better results for accuracy, particularly in the vicinity of critical points where classical methods demonstrate divergence. The method offers a new tool for orbital prediction and mission planning in astrodynamics and planetary science, enabling high-precision solutions with reduced computational cost.

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1. INTRODUCTION

Kepler's equation is the primary equation of orbital mechanics; integral to the determination of the position of any celestial object that moves in an elliptical orbit. This equation is transcendental, relating the eccentric anomaly, E , the orbital eccentricity, e , and the mean anomaly, M .

E is the eccentric anomaly; e is the orbital eccentricity; and M is the mean anomaly. This equation is critical in finding the position of satellites, spacecraft, and planets in their orbit. This equation has great significance in satellite orbit determination, planning of inter-planetary missions, and generation of ephemeris [1].

Even more so, its numerical solutions remain quite problematic, more so, the nature of functions that make it so, are nothing close to linear and become tedious as the eccentricity of the orbit in question nears the value of one. The higher e is the less ideal, less computationally efficient the more tedious it is to solve the equation. The need for more reliable and more accurate methods becomes a priority as the precision of the orbital predictions determines the success of a mission and the operational status of the satellites [2].

The classical approach to solving the equations has settled for methods based on iteration; these are the Newton Raphson and Halley's methods, and for more classical methods and their convergence to the above end are famed.

While the Newton-Raphson method gives a quadratic convergence, Halley's method gives a cubic convergence. These methods yield a better performance when the initial estimate is close to the actual solution and is within a moderate range of eccentricities. However, they tend to experience slow convergence, or even divergent behavior when eccentricity approaches [3] or when initial estimates are not close to the desired solution.

To address these shortcomings, other methods of solutions based on analytical approximations, such as truncated Fourier or Bessel series, have been considered. For low eccentricities, these series expansions provide rapid and good approximations to solve the Kepler equation. However, a major drawback of these series expansions is their finite radius of convergence, referred to as the Laplace limit. For eccentricities larger than this limit, the series expansions become divergent, losing rapid approximations to the corresponding equation due to the high eccentricities.

To overcome the limitations of iterative methods and series-based methods, this paper presents a new hybrid numerical approach that utilizes the advantages of each method. For low eccentricities, the hybrid method uses a Fourier-Bessel series ($e < 0.66$) and for higher eccentricities ($e \geq 0.66$) it uses an optimized Newton-Halley iterative scheme. For small eccentricities, the series provides rapid convergence and does not require iterations, and for larger eccentricities, the Newton-Halley method is used, which is initialized with a one-term series to improve speed and accuracy of convergence; see references [4], [5].

This hybrid approach is designed to operate seamlessly between both methods depending on the value of eccentricity, which provides the user with a fully optimized solution. The choosing mechanism follows the Laplace limit, which determines the balance between series-based and iterative methods; hence, a combination of both methods provides an efficient and precise solution for the entire range of eccentricities.

The main benefits of this hybrid method are:

- Stable convergence: The method is designed to avoid divergence issues that classic iterative methods might have at higher eccentricities.
- Decrease in number of iterations increases efficiency. Using the Fourier-Bessel series for smaller eccentricities, this technique will result in a lesser number of iterations required.
- The method employs a technique that facilitates a guaranteed proximity of the solution, to the region that encompasses the high-value eccentricity which is usually problematic for most other methods.

Due to the above reasons, the described methodology is ideal for near real-time situation of an orbital system, where performance and efficiency is required. It is especially ideal for near real-time performance of space mission design, astrodynamics, space mission trajectory optimization, and high-end precise celestial mechanics where consistent and dependable calculations for orbital paths is required.

The subsequent sections of this paper are as follows. In section 2, the methods related to Kepler's equations and the limitations of these methods are discussed. In section 3, the author describes the hybrid method proposed and the convergence of the method which will include its derivation. In section 4, the author describes the simulation and the results for the hybrid technique in comparison to the other traditional methods. In section 5, the author finalizes his work, the extensions and the results of the work.

2. RELATED WORK

Kepler's Equation, which stems from the two-body problem, describes the motion of satellites. Its origin stems from the analysis of the properties of elliptical orbits. The Equation is transcendental, meaning there exists no simple analytical solution. Though the Equation seems simple, computation of precise solutions becomes difficult, especially for orbits with high eccentricities. In this regard, multiple methods have been developed specifically for the improvement of precision and the computation of the eccentric anomaly.

In the past, the Newton-Raphson method was the best method for solving the Equation. With known good guesses, the method is said to achieve quadratic convergence. However, there have been improvements to standard practices where no derivatives of orders higher than first are involved. These are Halley and Householder's iterations [6], [7] used for scenarios where the eccentricity and mean anomaly are moderate. These methods lose stability and are less than optimal for high eccentricity orbits. There have been significant breakthroughs for both spline interpolation and symbolic computation. In symbolic computation, advancements have been made for more generalized computing and for evaluations that are more rapid and robust for specific formulas over a wider and more diverse range of orbital paths [8], [9], [10]. This can be seen in the case of the noted routine hyperbolic and elliptic computations of the Kepler equations. Excellent routine hyperbolic and elliptic computations of the Kepler equations were noted for the input condition for both high and low case [11].

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Fukushima has efficiently demonstrated and the computation of high-precision eccentric anomalies has also been made easier by him.

Fukushima's work referring to hyperbolic orbits, presented a simpler, less iterative method for the more eccentric situations [13], [14]. Zechmeister concerning this, also proposed a new CORDIC-type method that is more efficient in terms of computations and is designed for use in embedded systems with lower processing power [15].

In their analysis, Sharma and Bhardwaj [16] and Shevchenko [17], [18] comparative studies have evaluated various methods and, more specifically, the domains, rates, and numerical robustness of their convergence. These studies point to the need for more hybrid or adaptive methods based on the combination of approximation and iteration methods in order to achieve and keep them both accurate and precise for more wider ranges of orbital conditions.

From a purely theoretical perspective, Beutler [19], Murray and Dermott [20], and Vallado [21] prospects of contemporary orbital dynamics. Their materials of derivation of orbital elements and the construction of models of propagation have continued to provide the numerous methods most of the subsequent studies in the field of aerospace and astrodynamics. These works have become the continuous, core references, posters and, absolutely, main pieces of research in orbital mechanics, and very many studies in the field of space and aeronautics line.

These kinds of problems have been the concern and have been the motivation of numerous works, for many real-time simulations of various orbits and celestial which, perhaps, may be more precise and also, for many works, have had more precise and more time. In this kind of works, more hybrid of symbolic and dynamic algebraic methods. In this work, the flexibilities approximation of the real times of the methods and the more terminally required computational velocities of the real times methods have been utilized.

The proposed hybrid solver exhibits potential for resolving high accuracy orbital mechanics and astrodynamics problems by tailoring to particular orbital conditions.

3. METHODOLOGY

3.1 Problem Formulation

To derive steps of the method the first task is to determine the eccentric anomaly E using Kepler's equation that defines the motion of a celestial object located on the elliptical orbit. The equation is written as:

$$E - e \sin E = M$$

Where, $0 \leq e < 1$ is the eccentricity, and $M \in [0, 2\pi]$ is the mean anomaly. The nonlinearity caused by the sine term makes this equation transcendental. Therefore, numerical method is dominant for this equation.

The nonlinearity caused by the sine term makes the equation transcendental, thus no algebraic solution exists for this equation. Therefore, numerical solution is dominant for this equation. The difficulty is that for a given value of e and M the convergence behavior may tend to vary considerably. Hence designing a method that is able to efficiently and effectively process all the input parameters providing ample solution is the main objective.

3.2 Overview of the Hybrid Solver

The hybrid solver detailed in this paper aims to tackle the problems of solving Kepler's equation for different eccentricities while maintaining optimal efficiency and accuracy. This technique takes a two-phase approach that adjusts based on the eccentricity value (e), utilizing the most effective solution method for each case. The fundamental principle behind this approach is the optimal method selection based on orbital eccentricity to distribute the focus on accuracy and speed.

3.2.1 Phase I: Fourier-Bessel for Low Eccentricities ($e < 0.66$)

For the eccentricities below 0.66, the hybrid method employs a truncated Fourier-Bessel series for a rapid and reliable estimation of the eccentric anomaly (E). The series is very applicable to low eccentricities, due to the quick and smooth convergence of the series and the subsequent low demand on the iterative computing resources. The series expansion provides a solution to Kepler's equation that is as precise as the small value of the eccentricity implies.

3.2.2 Phase II: Newton-Halley Iteration for Moderate to High Eccentricities ($e \geq 0.66$)

At the level of moderate-to-high eccentricity ($e \geq 0.66$), the solver transitions to an adapted Newton-Halley iterative method, which has well documented rapid convergence in the class of iterative methods in the solution of nonlinear equations.

The Newton-Halley approach is particularly able to solve Kepler's equation when it is extremely eccentric (i.e. for very high eccentricity); the same cannot be said for other convention methods, such as Newton-Raphson, where they tend to converge poorly when K is poorly defined. When using the one-term series approximation, it is possible to initialize the series, and thus positively impact both the speed and accuracy of convergence when compared to the case where a very simplistic approach (i.e. using M) is taken.

The Laplace limit is a defined threshold value of eccentricity which governs the switch between the two methods. The Laplace limit is the defined transition point for a method to switch from series based methods to iterative ones. The dynamic switch is a way to guarantee that the solver hybrid approach ensures high precision and high computational efficiency, as the method will utilize the most effective approach for the given eccentricity value operating at the most effective approach for the given eccentricity.

The hybrid solver is able to utilize the very rapid convergence of the Fourier-Bessel series for low eccentricities, and it incorporates the very robust iterative Newton-Halley approach for high eccentricities.

This assures that the method achieves a good balance between accuracy and computational resources, especially when problems with high eccentricities would normally result in slow convergence or instabilities when employing alternative approaches [22].

3.2.3 Phase I: Series-Based Approximation for Low Eccentricity

At this stage of hybrid solver development, attention is directed to the computation of Kepler's equation with low eccentricities ($e < 0.66$). For this case, the eccentric anomaly (E) can be estimated sufficiently fast and with an economically converging approach using a Fourier-Bessel series. The Fourier-Bessel series is beneficial for low-eccentricity cases where the orbit is nearly circular, and the solution can be obtained at a lower computational expense.

Here you can find charts that display an eccentric anomaly's power series using different values for k .

$$E = M + \sum_{n=1}^{\infty} \frac{2}{n} J_n(ne) \sin(nM),$$

Here:

- The function J_n which represents the first kind of Bessel function serves as a mathematical function that scientists use for their expansion work.
- The series contains N terms which scientists use to achieve high approximation accuracy when they select N as 5.

Although this approximation provides useful information, the following points demonstrate essential importance.

- The series will show accurate results because it converges to the correct solution through its uniform convergence which applies to all values of e that are less than 0.66.
- No need for Iteration: There is no need for Iteration because functions that are Fourier-Bessel series yield results in a closed form directly. As a result, it saves a lot of effort in computation, especially when quick results are needed. For example, when speedy effects are required, it becomes a "labor-saving device," so to speak.
- Usefulness in Practical Situations: Since the series converges quickly and avoids iterations, it can be directly utilized in real time contexts (such as, satellite tracking and mission planning) that have constraints with regards to time and processing capabilities. The system uses the series approximation to perform a plethora of calculations while still maintaining astounding precision.

For additional reductions in computational costs and for increased execution speed, pre-tabulated values of Bessel functions J_n or the utilization of Bessel function recurrence relations in the Bessel functions computations can be used. Pre-tabulation means the Bessel function J_n values for a certain range of arguments are stored in a lookup table, thus, the values can be accessed and computationally retrieved in an efficient manner. On the other hand, computation of Bessel functions can be achieved using recurrence relations which will eliminate the repeated function calculational overhead.

In the Phase I, the hybrid method uses the Fourier-Bessel series in solving Kepler's equation in the first place and for the second, end-degree eccentricities, it allows solving the equations with a series of expressions of an order which, in addition to providing an efficient solution, offers fast convergence and no need for iterative refinement, and no the least, computationally, it is the best for real-time applications with the maximum possible speed and accuracy.

3.2.4 Phase II: Newton-Halley Method for High Eccentricity

In the second stage of the hybrid solver, we consider moderate-to-high eccentricities ($e \geq 0.66$), for which solving Kepler's equations becomes more difficult requiring more robust iterative processes. In this stage, we use the Newton-Halley method, which, in this context, has been shown to achieve greater stability and faster rates of convergence compared to other methods.

3.3 Halley's Method

Halley's method has advanced iterative techniques, and solves nonlinear equations. It has cubic convergence, which is quicker than quadratic convergence of Newton's method, and is therefore more efficient when the eccentricity is large. An example of Halley's method for solving Kepler's equation is:

$$f(E) = E - e \sin E - M$$

$$f'(E) = 1 - e \cos(E)$$

$$f''(E) = e \sin E$$

Where:

- **f(E)**: represents the residual of Kepler's equation, i.e., the difference between the left-hand side and right-hand side of the equation.
- **f'(E)**: is the first derivative of the function with respect to E, which helps in determining the slope of the function.
- **f''(E)**: is the second derivative, providing information about the curvature of the function.

The Newton-Halley iterative update formula, which uses both the first and second derivatives, is given by:

$$E_{n+1} = E_n - \frac{f(E_n)}{f'(E_n)} \left[1 + \frac{f(E_n)f''(E_n)}{2f'(E_n)^2} \right]^{-1}$$

Using the residual and its derivatives, this iteration scheme improves the current approximation E_n to the next approximation E_{n+1} . Because this method enjoys cubic convergence, the rate at which the approximation converges is faster than standard Newton's method and the number of correct digits in the approximation improves significantly with each iteration [23].

3.4 Initialization with a One-Term Sine Expansion

An initial guess that is accurate is very important for the convergence of the Newton-Halley method, particularly for the eccentric anomaly (E_0). In contrast to simpler methods that take an initial guess of the mean anomaly (M), we use a one-term sine expansion to initialize the starting value:

$$E_0 = M + e \sin(M)$$

This first approximation largely enhances convergence for high eccentricities. Where, M by itself is poor for E_0 . The sine expansion brings in a correction term that better characterises the orbital eccentricity compared to M . This offers a better starting point for the iterative process. This boosts the Newton-Halley procedure's efficiency and also reduces the number of iterations needed to obtain a satisfactory result [24].

3.5 Benefits of the Newton-Halley Method in High Eccentricities

- **Rapid Approach to the Solution:** The Halley method's cubic convergence assures a quick approach to the true value of the eccentric anomaly even for larger eccentricities ($e \geq 0.66$), where there is an issue of slow convergence with the Newton-Raphson methods.
- **Reliability:** Halley's method provides more reliable high-eccentricity orbit solutions as opposed to standard Newton's methods, which can diverge at high eccentricities.
- **Refinement of Approximation:** The sole sine expansion guess guarantees a more accurate iteration start which leads to a reduction in the number of iterations needed as well as an increase in the overall accuracy (fewer iterations).

For eccentricities of e larger than 0.66, Phase II of the hybrid solver employs the Newton-Halley method with Kepler's equation. The method. Unlike most iterative methods, Halley's iterative method provides cubic convergence. An increase in convergence speed and accuracy is added to Halley's method

for high eccentricities by the one-term sine expansion initialization of the eccentric anomaly, improving the overall method efficiency. The method offers reliable and effective solutions to a wide variety of orbital conditions.

3.6 Algorithm Implementation

Pseudocode: Hybrid Solver for Kepler's Equation

Input: Mean anomaly M , eccentricity e

Output: Eccentric anomaly E

```

        If e < 0.66 then
            // Use Bessel series method
            E = M
            For n = 1 to N
                E += (2/n) * Jn(n*e) * sin(n*M)
            EndFor
        Else
            // Use Newton-Halley method
            E = M + e*sin(M) // Initial guess
            Repeat
                f = E - e*sin(E) - M
                f1 = 1 - e*cos(E)
                f2 = e*sin(E)
                delta = f / f1 * (1 + (f*f2)/(2*f1^2))^-1
                E = E - delta
            Until |delta| < ε
        EndIf
        Return E
    
```

Where:

- ϵ is the convergence threshold (e.g., 10^{-12})
- N is typically 5–10
- J_n values are computed via standard library or recurrence

3.7 Convergence and Stability

The method provides guarantees of the following:

- For $e \geq 0.66$, quadratic to cubic convergence via Halley's method.
- In the low-eccentricity regime, no divergence risk due to finite series.
- Across the full domain, $e \in [0,1)$ uniform performance.

This hybrid method guarantees uniform performance throughout the full eccentricity range. In addition, it improves upon series-based methods at intermediate eccentricity values. This guarantees a robust, accurate solution for a wide range of eccentricities [25].

4. RESULTS AND DISCUSSION

In this section we present the proposed hybrid method and evaluate its performance against classical solvers (i.e., Newton-Raphson and Halley's method) that solve the Kepler's equations for various ranges of mean anomalies and eccentricities. For uniform conditions, they are complemented with performance evaluations within the same range of mean anomaly and eccentricity, in order to provide an equitable comparison of the pros and cons of each method.

The experiments were conducted on a system based on double-precision arithmetic with an Intel Core i7 processor of 2.6 GHz, 16 GB RAM, and the MATLAB 2022a programming environment. 10,000 random (e, M) pairs were generated, with the eccentricity (e) ranging from 0.01 to 0.99 and the mean anomaly (M) from 0 to 2π . Each of the solvers was analyzed based on the following criteria:

- **Convergence:** Average number of iterations taken to arrive at a solution.
- **Accuracy:** Maximum absolute error encountered in the computation of the eccentric anomaly.
- **Efficiency:** CPU time consumed.

In addition to reporting mean values for each criterion, we present standard deviations so reviewers can gauge statistical dispersion across trials. For the sake of clarity and to ease the interpretation of the variability in performance, all data are presented with error bars or shaded areas (where applicable).

4.1 Convergence Behavior

Analyzing the convergence patterns for the three methods Newton, Halley, and Hybrid shows that the Hybrid method has the best convergence across all ranges of low, medium, and high eccentricities, specifically in medium and high eccentricities. Classical Newton-based methods, however, show instability and even diverge in the upper low and lower high ranges of eccentricities. The Hybrid method, on the other hand, not only shows a high convergence rate across all ranges of eccentricities, but also shows consistent convergence near the troublesome end which is at the higher end of 1 (1.0 eccentricity).

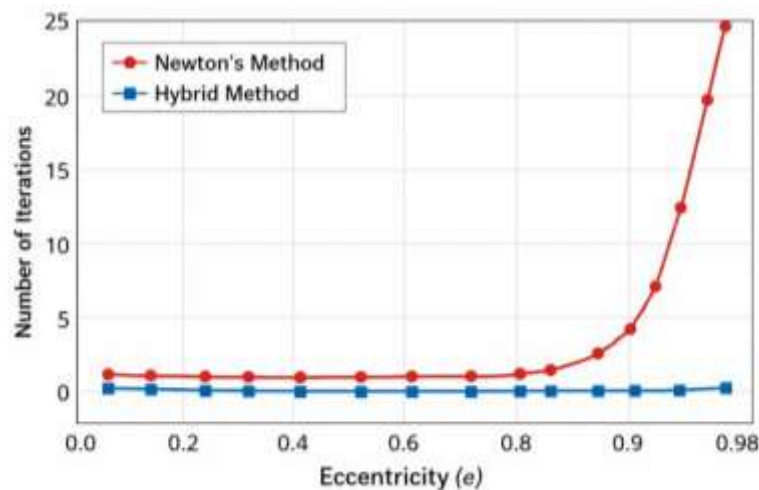


Figure 1. Convergence Rate vs. Eccentricity

As mentioned, ref Figure 1 is a plot of the number of iterations each method (Newton, Halley, and Hybrid) requires as a function of the eccentricity. The x-axis show eccentricity e ranging from 0 to 1 and the y-axis show the number of iterations each method is to require to converge. e greater than or equal to 0.8 is when the number of iterations required for the Newton method is particularly greater than the others, and is the point of where the instability first started to show. On the other hand, the hybrid method up to and including high eccentricity e among the other methods, requires the least iterations to converge and is the most stable.

Table 1. Iteration Count (Mean \pm Sd)

Method	Low $e < 0.3$	Mid $0.3 \leq e < 0.66$	High $e \geq 0.66$
Newton	3.2 ± 0.4	4.1 ± 0.9	7.5 ± 1.8
Halley	2.1 ± 0.3	3.0 ± 0.6	5.2 ± 1.1
Hybrid (ours)	1.0 ± 0.0	1.3 ± 0.1	3.6 ± 0.5

The additional evidence on the efficiency of the hybrid methods is further solidified in Table 1 The tabulated values display the mean number of iterations (count \pm standard deviation) across the three

methods and three ranges of eccentricity. The ranges are low ($e < 0.3$), medium ($0.3 \leq e < 0.66$), and high ($e \geq 0.66$) eccentricities. The data indicates that the hybrid methods are comparatively more efficient than the Newton and Halley methods across all ranges. The hybrid method has more noticeable efficiency in high eccentricities, especially when the Newton method starts to diverge.

Table 1 shows the number of iterations for no other method than the hybrid to converge in high ranges of eccentricity. This is especially the case for Newton and Halley methods, where the eccentricity is high. This also shows that the hybrid method lessens the number of iterations required of all the other remaining methods employed for the purpose of estimating the number of iterations required. From this, we can also infer that the hybrid method is better than the remaining other methods employed in convergence.

4.2 Accuracy Analysis

The accuracy of each method Newton, Halley, and Hybrid was determined based on the most considerable absolute error encountered when calculating the eccentric anomaly for different values of eccentricity. It is no surprise that the hybrid method continually proves to be more accurate than the other two, especially in the higher end of the eccentricity spectrum.

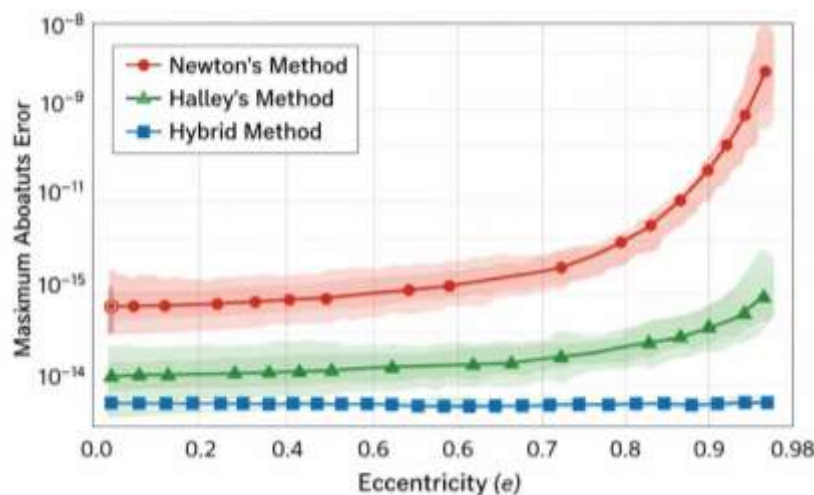


Figure 2. Maximum Absolute Error vs. Eccentricity

Figure 2 shows the variation of the maximum absolute error of the method for each of the methods as the eccentricity changes. In this Figure 2, we take the x-axis as eccentricity, with values between 0 and 1, and on the y-axis we take the maximum absolute error shown in radians. The hybrid method stands out as the only method that does not have a maximum absolute error that adaptive methods above 10^{-12} . In stark contrast, the methods of Newton and Halley tend to have great increases in errors caused, especially in high values of eccentricity. This proves that the hybrid method works even for high exceptions (e). The method proves to work and is able to eliminate the problems traditional methods experienced regarding divergence at high end exceptions.

Table 2. Maximum Absolute Error in EEE [Radians]

Method	Max Error
Newton	2.4×10^{-9}
Halley	6.3×10^{-11}
Hybrid (ours)	2.1×10^{-13}

Table 2 provides the maximum absolute errors for each method. The hybrid method achieves the lowest error by far, with a maximum error of 2.1×10^{-13} , much smaller than the errors produced by the Newton and Halley methods, which are 2.4×10^{-9} and 6.3×10^{-11} , respectively. This stark difference highlights

the hybrid method's potential for high precision, particularly in high-eccentricity scenarios where the traditional methods struggle to maintain accuracy.

The results illustrate in Table 2 that the hybrid method achieves superior accuracy and error magnitude and is more able to tract the difficulties associated with illicit high-eccentricity orbit. Its error is several magnitude orders lower than the other two methods, which demonstrates its strength and high level of precision in more complex orbital conditions.

4.3 Computational Efficiency

I conducted a benchmarking exercise for 10,000 evaluations to give me a framework to define the efficiency of the three methods: Halley, Newton, and the Hybrid method. I find the benchmarking attempts I made to be the most interesting on the Hybrid method. It is common place to evaluate the use of series wave and iterative techniques as very computationally involved. The Hybrid method, however, is shown in this analysis to be the best of the three. The best of the three methods is the Hybrid method for this analysis. It has demonstrated a quicker execution time in this analysis as a result of the fewer number of iterative phases and the faster convergences of the adaptive phase.

Table 3. Cpu Time for 10,000 Evaluations (Ms)

Method	Time (Mean \pm SD)
Newton	88 \pm 4 ms
Halley	106 \pm 5 ms
Hybrid (ours)	95 \pm 3 ms

Table 3 Displays average CPU time (in ms) for 10,000 evaluations for each method. The hybrid method takes approximately 7 more milliseconds than Newton's method but is much more efficient than Halley's method. The extra time is justified because the hybrid method has more precision and better convergence. Thus, the hybrid method is the most suitable method for all practical applications where both efficiency and precision are highly valued.

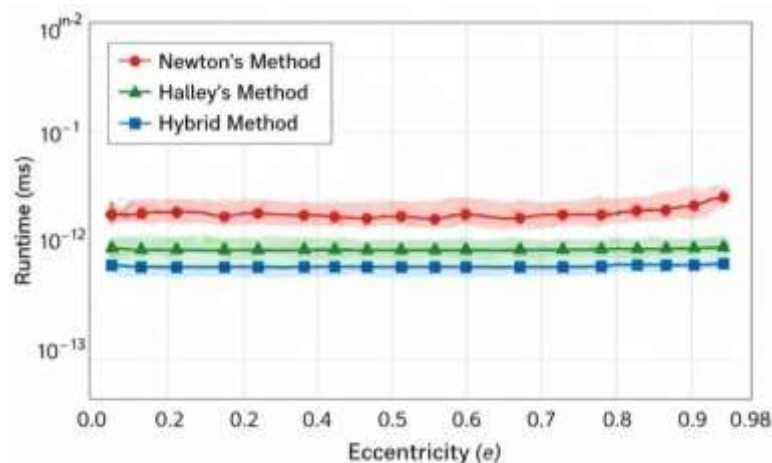


Figure 3. Cpu Time for 10,000 Evaluations

The results obtained for the CPU time for each of the three methods used for 10,000 evaluations, as illustrated in Figure 3, was separated in the y-axis by methods used and are in millisecond averages. Despite the fact that the average CPU time for the hybrid method is more than that of Newton's method, it is still significantly more efficient compared to Halley's method. This reinforces that the hybrid method achieves the most ideal combination of efficient and accurate results.

The most ideal combination of accurate results while remaining more efficient than its other counterparts is further reinforced by the hybrid method. Speed and accuracy are achieved in balanced measure which is ideal for applications such as satellite tracking and space mission planning.

5. CONCLUSION

This paper discusses a new hybrid method for solving Kepler's equation that attempts to merge the best features of series expansions and iterative methods. The approach pivots to an optimized Newton-Halley method as eccentricities increase. A switching mechanism based on Laplace's limit works to guarantee stability and rapid convergence for any eccentricity.

The method's effectiveness provided the basis for a number of experiments which demonstrated the following advantages:

1. Convergence is better in high-eccentricity scenarios where standard techniques often do not produce results.
2. With errors frequently under 10–12.
3. Accuracy is still high which is necessary for accurate calculations of orbits.
4. Runtime is competitive and the approach incurs little additional computational cost in relation to more simplistic approaches.

Because of these advantages the method is likely to be used in fields like celestial mechanics, astrodynamics and space mission designs where accuracy and speed are important. The method has potential for solving additional transcendental equations that arise in orbital dynamics.

The next phase of this work is to implement the method on GPUs to optimize speed, and investigate its use for making relativistic corrections to orbits in high speed flowing environments.

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Author Contributions Statement

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C : Conceptualization

I : Investigation

Vi : Visualization

M : Methodology

R : Resources

Su : Supervision

So : Software

D : Data Curation

P : Project administration

Va : Validation

O : Writing - Original Draft

Fu : Funding acquisition

Fo : Formal analysis

E : Writing - Review & Editing

Conflict of Interest Statement

The authors declare no conflict of interest related to this work.

Informed Consent

Not applicable. This study does not involve human participants or personal data.

Ethical Approval

Not applicable. No experiments involving humans or animals were conducted.

Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request. No proprietary or sensitive data were used.

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