Journal of Energy Engineering and Thermodynamics
ISSN 2815-0945
Vol: 04, No. 04, June-July 2024
https://journal.hmjournals.com/index.php/JEET
DOI: https://doi.org/10.55529/jeet.44.10.20

Analyzing Convergence and Accuracy of Load Flow Methods in the IEEE 39-Bus Power System

Md Al Araf[1*], Saiful Islam Tuhin[2], Shaneen Ara[3], Mohammad Tarikul Islam Rimon[4], M. Mohi Uddin[5]

[1,2,4,5]Department of EEE, BUBT, Dhaka 1216, Bangladesh.*
[3] Department of CSE, BUBT, Dhaka 1216, Bangladesh.

Email: [2]saifulislamtuhin15@gmail.com, [3]shaneenara88@gmail.com,
[4]tarikulrimon2@gmail.com, [5]mohiuddinhasan98@gmail.com
Corresponding Email: [1]mdalaraf@icloud.com*

Received: 25 February 2024 **Accepted:** 11 May 2024 **Published:** 26 June 2024

Abstract: This research paper presents a comprehensive comparative analysis of load flow methods applied to the IEEE 39 bus system, focusing on Newton-Raphson, Gauss-Seidel, and two variations of the Fast Decoupled method, XB and BX. The study evaluates the convergence behavior and iteration performance of these methods across two widely used software platforms: MATLAB and Power World Simulator. Through extensive simulations, the research investigates the convergence characteristics and computational efficiency of each method, providing insights into their respective advantages and limitations. The findings offer valuable guidance for power system engineers and researchers in selecting suitable load flow methods for analyzing complex power systems.

Keywords: Load Flow Methods, Newton-Raphson, Gauss-Seidel, Fast Decoupled, IEEE 39 Bus System, Convergence Characteristics.

1. INTRODUCTION

The analysis of power flow in electrical networks is fundamental for the efficient operation and planning of modern power systems [1]. There are many ways for solving power flow equations but the Newton-Raphson, Gauss-Seidel, and Fast Decoupled methods [2] stand out as prominent techniques widely used by researchers and engineers. These methods are very important to determine the steady-state operating conditions of power systems, ensuring stability and reliability [3]. The IEEE 39 bus system, a standard benchmark for power system studies, provides a realistic representation of a medium-sized power network. Through the application of different load flow methods to this system, researchers gain valuable insights into the performance and behavior of these algorithms under various operating conditions. Furthermore, comparing the convergence behavior and computational efficiency of these methods across different software platforms such as MATLAB and Power World Simulator enhances our understanding of their practical applicability and scalability. This research paper presents a thorough investigation into the Newton-Raphson, Gauss-Seidel, and Fast Decoupled methods applied to the IEEE 39 bus system [4]. By analyzing their convergence characteristics and computational performance, this study aims to provide a comprehensive comparison of these methods. Additionally, by utilizing two distinct software platforms, MATLAB and Power World Simulator, this research offers insights into the impact of software implementation on the performance of these methods.

2. RELATED WORK

Many studies have addressed power flow analysis methods and their applications in power systems. Chen et al. (1991) proposed a rigorous approach for three-phase distribution systems using an optimally oriented triangular factorization with Y/ sub BUS/ method [2]. This method can handle balanced/ unbalanced, network, radial or mixed-type systems. Slochanal et al. (2005) introduced a load flow analysis method for systems integrated with Unified Power Flow Controllers (UPFCs), highlighting its ease of use for obtaining equivalent networks and determining UPFC control parameters [5]. Jangra and Vadhera (2017) compared the results from the Newton-Raphson method applied to unbalanced radial distribution feeders in MATLAB/Simulink with solutions from the IEEE distribution system analysis subcommittee [6]. Huiping et al. (2018) introduced a critical line identification method using probabilistic load flow analysis, considering uncertainties in wind power and load [7]. Focusing on the system resilience, Russell and Khan (2022) investigated the impact of single line outages using Contingency Analysis on the IEEE 39-bus system [8]. Their study showed effects on various aspects including generator and voltage constraints, transmission line loading, and islanding potential. Most recently, Tricarico et al. (2023) presented a modified IEEE 39-bus system tailored for zonal day-ahead market (ZDAM) simulations, emphasizing the integration of renewable energy sources like solar and wind power [9]. Their work highlights the importance of studying power flow under ZDAM conditions. In this study looks at how Newton-Raphson, Gauss-Seidel, and Fast-Decoupled are used to analyze power flow in the IEEE 39-bus system. By looking at how well they work on various software systems, the study gives us useful information about how well they work in real-life situations. This study is another addition to the increasing amount of work on methods for power flow analysis, which are very important for making sure that modern power systems work efficiently and reliably.

3. METHODOLOGY

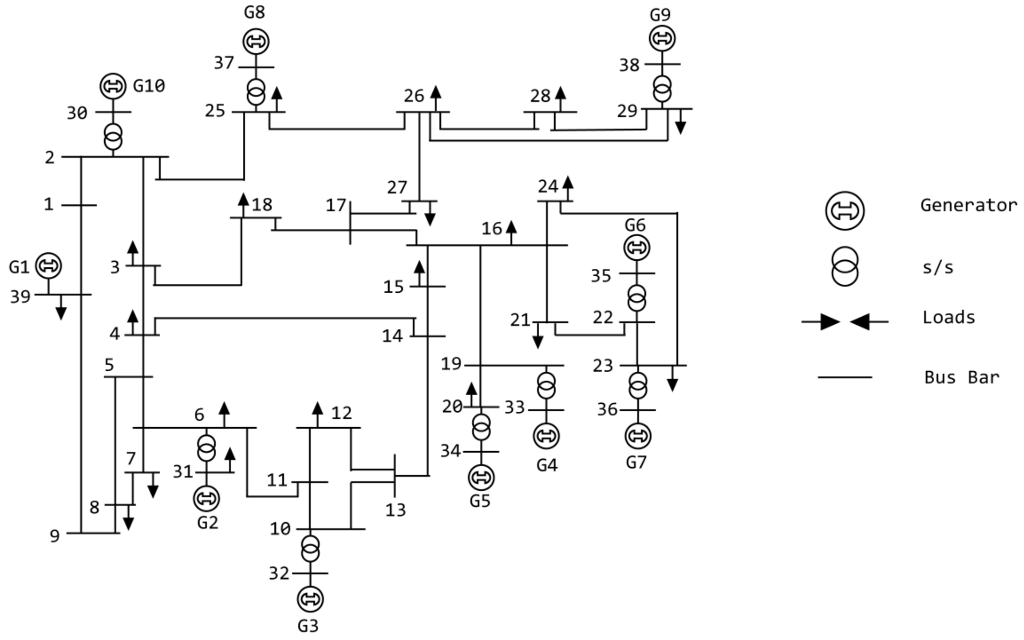
Load flow analysis, sometimes referred to as power flow analysis, is an essential methodology employed in the field of power system research for the purpose of ascertaining the stable operational circumstances of electrical networks. The main goal of load flow analysis is to calculate the voltage magnitudes, phase angles, and power flows at all bus positions in the network, based on the system's structure, load requirements, and generation schedules. The load flow equations are nonlinear algebraic equations that explain the balance of active and reactive power at each bus in the network, subject to voltage magnitude and phase angle limitations. Mathematically, the load flow equations can be expressed as follows:

$$P_i = \sum_{j=1}^{n} V_i V_j \left(G_{ij}cos(\theta_{ij}) + B_{ij}sin(\theta_{ij})\right) \qquad (1)$$

$$Q_i = \sum_{j=1}^{n} V_i V_j \left(G_{ij}sin(\theta_{ij}) - B_{ij}cos(\theta_{ij})\right) \qquad (2)$$

Where P_i and Q_i are the active and reactive power inputs at bus i, respectively. V_i and θ_i are the voltage level and phase angle at bus i, respectively. G_{ij} and B_{ij} represent the real and imaginary components of the admittance between bus i and j, respectively, n is the total number of buses in the network. Solving these nonlinear equations requires iterative numerical methods, among which the Newton-Raphson, Gauss-Seidel, and Fast Decoupled methods are the most commonly used [10], [11], [12].

This section explores the strategies used to solve load flow equations in power system, specifically focusing on three well-known iterative techniques: Newton-Raphson, Gauss-Seidel and Fast Decoupled. The Newton-Raphson method [13] is a fundamental methodology that uses a Jacobian matrix to transfer nonlinear equations into linear ones. This allows for quick convergence in systems that are well conditioned. Nevertheless, it may encounter difficulties with divergence or sluggish convergence in situations that are ill conditioned or extremely nonlinear [14]. The Guass-Seidel technique [15] employs a sequential updating mechanism, which simplifies implementation but may result in slower convergence or oscillatory behavior in complicated systems. The fast Decoupled method [16] which is a modified version of the Newton-Raphson method, enhances computational efficiency and



accuracy by reducing the Jacobian matrix. This characteristic makes it especially suitable for conducting comprehensive examination of power systems on a broad scale.
Figure 1: Single line diagram of IEEE 39 Bus System

Simulation Setup and Parameters
The analysis of the IEEE 39 bus system using MATLAB leveraged the capabilities of the MatPower toolbox, which offers a suite of functions for solving power flow equations, encompassing different methods. The IEEE 39 bus case data, encompassing bus details, generation specifics, load characteristics, and branch parameters, were imported into MATLAB for analysis.

The analytical process comprised the following steps:
1. Data Input: MATLAB was employed to input the comprehensive IEEE 39 bus case data, encompassing bus, generation, load, and branch particulars.
2. Power Flow Analysis: Utilizing Mat Power's functions, the power flow equations were meticulously solved, yielding voltage magnitudes and angles for each bus in per unit, along with generation and load metrics in MW and MVAr units. Additionally, branch details such as line flows and losses were computed.

Bus No.	**Voltage**		**Generation**		**Load**	
	Magnitude (Pu)	Angle (Degree)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)
1	1.039	-13.537	-	-	97.60	44.20
2	1.048	-9.785	-	-	-	-
3	1.031	-12.276	-	-	322.00	2.40
4	1.004	-12.627	-	-	500.00	184.00
5	1.006	-11.192	-	-	-	-
6	1.008	-10.408	-	-	-	-
7	0.998	-12.756	-	-	233.80	84.00
8	0.998	-13.336	-	-	522.00	176.60
9	1.038	-14.178	-	-	6.50	-66.60
10	1.018	-8.171	-	-	-	-
11	1.013	-8.937	-	-	-	-
12	1.001	-8.999	-	-	8.53	88.00
13	1.015	-8.930	-	-	-	-
14	1.012	-10.715	-	-	-	-
15	1.016	-11.345	-	-	320.00	153.00
16	1.033	-10.033	-	-	329.00	32.30
17	1.034	-11.116	-	-	-	-
18	1.032	-11.986	-	-	158.00	30.00
19	1.050	-5.410	-	-	-	-
20	0.991	-6.821	-	-	680.00	103.00

21	1.032	-7.629	-	-	274.00	115.00
22	1.050	-3.183	-	-	-	-
23	1.045	-3.381	-	-	247.50	84.60
24	1.038	-9.914	-	-	308.60	-92.20
25	1.058	-8.369	-	-	224.00	47.20
26	1.053	-9.439	-	-	139.00	17.00
27	1.038	-11.362	-	-	281.00	75.50
28	1.050	-5.928	-	-	206.00	27.60
29	1.050	-3.170	-	-	283.50	26.90
30	1.050	-7.370	250.00	161.76	-	-
31	0.982	0.000*	677.87	221.57	9.20	4.60
32	0.984	-0.188	650.00	206.96	-	-
33	0.997	-0.193	632.00	108.29	-	-
34	1.012	-1.631	508.00	166.69	-	-
35	1.049	1.777	650.00	210.66	-	-
36	1.064	4.468	560.00	100.16	-	-
37	1.028	-1.583	540.00	-1.37	-	-
38	1.026	3.893	830.00	21.73	-	-
39	1.030	-14.535	1000.00	78.47	1104.00	250.00
Total			**6297.87**	**1274.94**	**6254.23**	**1387.10**

Table 1: Bus data from MATLAB using four methods

Branch	**From Bus**	**To Bus**	**From Bus Injection**		**To Bus Injection**		**Loss (I^2Z)**	
			P (MW)	Q (MVAr)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)
1	1	2	-173.70	-40.31	174.68	-24.36	0.978	11.48
2	1	39	76.10	-3.89	-76.03	-74.75	0.066	1.65
3	2	3	319.91	88.59	-318.58	-100.88	1.335	15.51
4	2	25	-244.59	82.97	248.93	-93.84	4.337	5.33
5	2	30	-250.00	-147.20	250.00	161.76	0.00	14.56
6	3	4	37.34	113.06	-37.13	-132.59	0.208	3.40
7	3	18	-40.76	-14.59	40.78	-7.94	0.017	0.21
8	4	5	-197.45	-4.09	197.76	-4.52	0.309	4.95
9	4	14	-265.42	-47.32	265.99	42.48	0.571	9.22
10	5	6	-536.94	-43.11	537.51	46.16	0.573	7.45
11	5	8	339.18	47.64	-338.24	-49.39	0.933	13.07
12	6	7	453.82	81.55	-452.56	-73.59	1.261	19.33
13	6	11	-322.65	-38.85	323.38	33.14	0.724	8.48
14	6	31	-668.67	-88.85	668.67	216.97	0.00	128.12
15	7	8	218.76	-10.41	-218.56	4.84	0.192	2.21
16	8	9	34.81	-132.06	-34.48	97.72	0.324	5.11
17	9	39	27.98	-31.12	-27.97	-96.78	0.018	0.44

Journal of Energy Engineering and Thermodynamics
ISSN 2815-0945
Vol: 04, No. 04, June-July 2024
https://journal.hmjournals.com/index.php/JEET
DOI: https://doi.org/10.55529/jeet.44.10.20

18	10	11	327.90	73.37	-327.46	-76.18	0.438	4.71
19	10	13	322.10	37.49	-321.69	-40.65	0.407	4.38
20	10	32	-650.00	-110.87	650.00	206.96	0.00	96.10
21	12	11	-4.06	-42.25	4.09	43.04	0.029	0.79
22	12	13	-4.47	-45.75	4.51	46.68	0.034	0.93
23	13	14	317.18	-6.03	-316.30	-1.80	0..879	9.87
24	14	15	50.31	-40.68	-50.26	3.66	0.053	0.64
25	15	16	-269.74	-156.66	270.56	147.33	0.825	8.61
26	16	17	224.02	-42.54	-223.68	32.50	0.338	4.29
27	16	19	-451.30	-54.20	454.38	58.75	3.078	37.52
28	16	21	-329.60	14.44	330.42	-27.74	0.821	13.86
29	16	24	-42.68	-97.33	42.71	90.63	0.030	0.59
30	17	18	199.04	11.05	-198.78	-22.06	0.261	3.06
31	17	27	24.64	-43.56	-24.62	9.23	0.016	0.21
32	19	20	174.73	-9.17	-174.51	13.48	0.218	4.30
33	19	33	-629.11	-49.58	632.00	108.29	2.894	58.71
34	20	34	-505.49	-116.48	508.00	166.69	2.511	50.21
35	21	22	-604.42	-87.26	607.21	108.15	2.783	48.70
36	22	23	42.79	41.88	-42.77	-61.75	0.025	0.40
37	22	35	-650.00	-150.04	650.00	210.66	0.00	60.63
38	23	24	353.84	-0.50	-351.31	1.57	2.529	40.24
39	23	36	-558.57	-22.35	560.00	100.16	1.430	77.82
40	25	26	65.41	-18.81	-65.29	-39.04	0.126	1.27
41	25	37	-538.34	65.45	540.00	-1.37	1.657	64.08
42	26	27	257.30	68.21	-256.38	-84.73	0.920	9.66
43	26	28	-140.19	-21.21	141.61	-56.36	0.788	8.69
44	26	29	-190.19	-24.96	192.10	-67.79	1.914	20.98
45	28	29	-347.61	28.76	349.16	-39.44	1.556	16.78
46	29	38	-824.77	80.33	830.00	21.73	5.234	102.06
Total:							**43.641**	**1000.59**

Table 2: Branch data of IEEE 39 Bus system using all four methods

In parallel with MATLAB analysis, the IEEE 39 bus system underwent scrutiny utilizing Power World Simulator, a commercial software renowned for its efficacy in power system simulation. The Power World Simulator analysis encompassed the following steps:

1. Load Flow Diagram: An exhaustive load flow analysis was executed in Power World Simulator to ascertain the steady-state operational parameters of the IEEE 39 bus system. This analysis provided a graphical representation of voltage magnitudes, phase angles, and power flows within the network.
2. Simulation: Diverse simulations were undertaken to evaluate the system's performance under varied operational scenarios, including contingency, voltage stability, and transient stability

analyses. These simulations facilitated an assessment of system reliability and the identification of potential vulnerabilities.

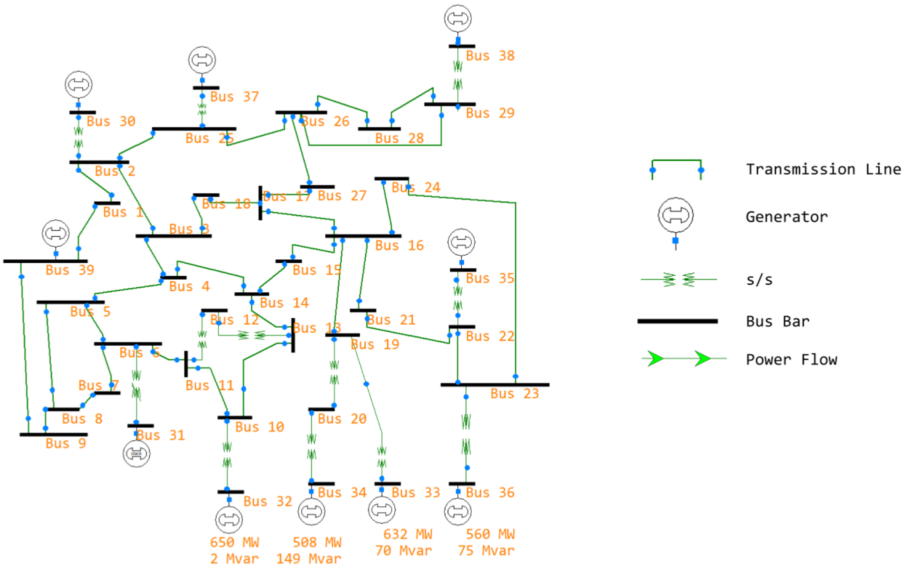


Figure 2: Load flow diagram in Power World Simulator

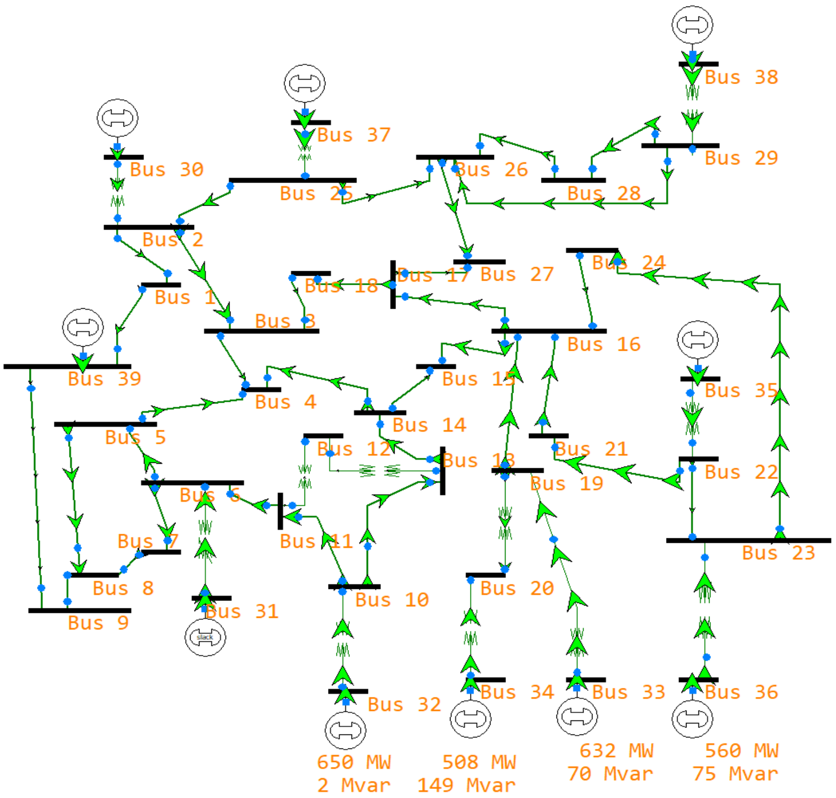


Figure 3: Load flow simulation in Power World Simulator

Journal of Energy Engineering and Thermodynamics
ISSN 2815-0945
Vol: 04, No. 04, June-July 2024
https://journal.hmjournals.com/index.php/JEET
DOI: https://doi.org/10.55529/jeet.44.10.20

4. RESULTS AND DISCUSSION

In the analysis of the IEEE 39 bus system, a comprehensive array of parameters was scrutinized to gain insights into its operational characteristics. The system comprises 39 buses interconnected through 46 branches, with a diverse generation mix including 10 generators with a total capacity of 7367.0 MW. These generators, committed for operation, demonstrated an actual generation of 6297.9 MW and 1274.9 MVAr. Conversely, the 21 loads, with a combined demand of 6254.2 MW and 1387.1 MVAr, illustrated the load distribution across the network. While fixed loads constituted the majority, dispatchable loads were absent. The absence of shunts indicated a balanced injection of reactive power. Additionally, the system featured 12 transformers and 6 inter-ties, facilitating connectivity and power exchange with neighboring networks. Analysis of voltage magnitudes revealed notable variations, with a minimum of 0.982 p.u. at bus 31 and a maximum of 1.064 p.u. at bus 36. Voltage angles exhibited variability, ranging from 14.54 degrees at bus 39 to 4.47 degrees at bus 36. Notably, active power losses amounted to 5.23 MW, while reactive power losses reached 128.12 MVAr across specific branches, underscoring the significance of network efficiency and stability considerations in power system analysis.

Numbers		**Parameters**	**P (MW)**	**Q (MVAr)**
Buses	39	Total Gen Capacity	7367.0	-160.0 to 2807.0
Generators	10	On-line Capacity	7367.0	-160.0 to 2807.0
Committed Generators	10	Generation (Actual)	6297.9	1274.9
Loads	21	Load	6254.2	1387.1
Fixed	21	Fixed	6254.2	1387.1
Dispatchable	0	Dispatchable	-0.0 of -0.0	-0.0
Shunts	0	Shunt (inj)	-0.0	0.0
Branches	46	Losses (I^2Z)	43.64	1000.59
Transformers	12	Branch Charging (inj)	-	1112.8
Inter-ties	6	Total Inter-tie Flow	719.8	256.9
Areas	3			

Parameter	**Minimum**	**Maximum**
Voltage Magnitude	0.982 pu at Bus 31	1.064 pu at Bus 36
Voltage Angle	-14.54 degree at Bus 39	4.47 degree at Bus 36
P Losses (I^2R)	-	5.23 MW at line 29-38
Q Losses (I^2X)	-	128.12 MVAr at line 6-31

Table 3: Parameters from MATLAB Toolbox

The examination of the IEEE 39 bus system utilizing diverse power flow techniques yielded insightful findings, elucidating the system's operational dynamics and computational efficacy. Three distinct methodologies were applied: Newton-Raphson, Gauss-Seidel, and Fast-decoupled. Each approach manifested unique attributes concerning convergence behavior and computational efficiency. The Newton-Raphson method displayed the swiftest convergence, necessitating only a single iteration for solution attainment, coupled with a minimal computational time of 0.02 seconds. Conversely, the Gauss-Seidel method entailed a higher iteration count (66), albeit achieving convergence within a reasonable time frame of 0.04 seconds. The Fast-decoupled method, in both BX and XB formulations, demonstrated competitive convergence rates alongside minimal computational time, rendering it a feasible choice for comprehensive analysis of large-scale power systems. In summation, these outcomes underscore the significance of method selection predicated on computational efficiency and convergence properties to ensure precise and timely power system analysis. According to the Table 1, various power flow methods assessed, the Newton-Raphson Method for AC Power Flow emerges as the most efficient, requiring just a single iteration to attain convergence with a computational time of merely 0.02 seconds.

Method	Type of Power Flow	Iterations	Converge Timing
Newton Raphson Method	AC Power Flow	1	0.02 seconds
Gauss-Seidel Method		66	0.04 seconds
Fast-decoupled, BX		4 P-iterations	0.02 seconds
Fast-decoupled, XB		3 Q-iterations	0.03 seconds

Table 4: Comparison of iterations and converging times on different methods according to AC Power Flow.

Its rapid convergence and minimal computational burden showcase its superiority in power flow analysis. Following closely is the Fast-decoupled Method (BX), which, despite necessitating four P-iterations and three Q-iterations, achieves convergence swiftly within 0.02 seconds, demonstrating commendable performance. The Fast-decoupled Method (XB) also displays efficiency, achieving convergence with a computational time of 0.03 seconds. In contrast, the Gauss-Seidel Method for AC Power Flow lags behind, requiring 66 iterations and 0.04 seconds of computational time to achieve convergence, positioning it as the least efficient among the evaluated methods. Overall, the Newton-Raphson Method stands out as the most effective solution for AC Power Flow analysis, surpassing both fast-decoupled methods and the Gauss-Seidel Method in terms of iteration efficiency and computational time.

5. CONCLUSION

Throughout this study, a thorough examination of load flow methods employed in power system analysis, specifically the Newton-Raphson method, Gauss-Seidel method, and Fast Decoupled method, has been conducted. These methodologies serve as fundamental tools in establishing the steady-state operational parameters of power systems, crucial for efficient energy transmission and distribution. The Newton-Raphson method stands out for its notable capacity for rapid convergence in well-conditioned systems, making it a robust solution for addressing the nonlinear equations inherent in load flow analysis. However, the existence of convergence challenges, particularly in ill-conditioned or highly nonlinear systems, underscores the imperative for further research to broaden its applicability across diverse contexts. Conversely, the Gauss-Seidel method, characterized by its sequential approach to updating state variables, offers a simplified and readily implementable solution. Yet, its susceptibility to slow convergence or oscillatory behavior in highly nonlinear systems necessitates ongoing exploration of algorithmic enhancements to bolster its efficacy. The Fast Decoupled method, a derivative of the Newton-Raphson method, strikes a balance between computational efficiency and accuracy by streamlining computations while preserving requisite precision. Its effectiveness in handling large-scale power systems has propelled its widespread adoption. Nonetheless, continual refinement and optimization efforts are indispensable for maximizing its potential. Looking ahead, future trajectories in load flow analysis may entail the integration of advanced optimization techniques, such as machine learning algorithms, to augment convergence rates and robustness across diverse system conditions. Again, as renewable energy sources and distributed generation gain prominence, the development of load flow methods capable of accommodating their distinct characteristics and uncertainties becomes imperative. There exists an avenue for exploration in the integration of Simulink and other analytical tools to bolster the capabilities of load flow analysis. Optimization techniques like Ant Colony Optimization (ACO), Bee Colony Optimization (BCO), Particle Swarm Optimization (PSO), and Bacterial Foraging Optimization (BFO) hold promise for enhancing the efficiency and efficacy of power system analysis. Future research could delve into areas such as optimal power flow, network reconfiguration, and economic load dispatch to address emerging challenges such as grid resilience, cybersecurity, and real-time operation in interconnected smart grids. Collaborative endeavors between academia, industry, and regulatory bodies are paramount for advancing the frontier of load flow analysis, ensuring the reliability and efficiency of power systems in the dynamic landscape of energy provision.

Declaration

Funding: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Competing Interests: The authors declare that they have no competing interests.
Ethics Approval: Not applicable
Consent for Publication: The authors declare that all authors consented to the publication of this research and the included data.
Data Availability: The data used in this study are available from the authors upon reasonable request.

6. REFERENCES

1. X. Zhang, Ed., Restructured Electric Power Systems: Analysis of Electricity Markets with Equilibrium Models, 1st ed. Wiley, 2010. Doi: 10.1002/9780470608555.
2. T.-H. Chen, M.-S. Chen, K.-J. Hwang, P. Kotas, and E. A. Chebli, “Distribution system power flow analysis-a rigid approach,” IEEE Trans. Power Deliv., vol. 6, no. 3, pp. 1146–1152, Jul. 1991, doi: 10.1109/61.85860.
3. A. A. Eajal, M. A. Abdelwahed, E. F. El-Saadany, and K. Ponnambalam, “A Unified Approach to the Power Flow Analysis of AC/DC Hybrid Microgrids,” IEEE Trans. Sustain. Energy, vol.7, no. 3, pp. 1145–1158, Jul. 2016, doi: 10.1109/TSTE.2016.2530 740.

4. H. Kaur, Y. S. Brar, and J. S. Randhawa, “Optimal power flow using power world simulator,” in 2010 IEEE Electrical Power & Energy Conference, Halifax, NS, Canada: IEEE, Aug. 2010, pp. 1–6. Doi: 10.1109/EPEC.2010.5697188.
5. S. M. R. Slochanal, S. Latha, and K. Chithiravelu, “A novel approach of power flow analysis incorporating UPFC,” in 2005 International Power Engineering Conference, Singapore: IEEE, 2005, pp. 701-704 Vol. 2. Doi: 10.1109/IPEC.2005.206998.
6. J. Jangra and S. Vadhera, “Load flow analysis for three phase unbalanced distribution feeders using Matlab,” in 2017 2nd International Conference for Convergence in Technology (I2CT), Mumbai: IEEE, Apr. 2017, pp. 862–866. Doi: 10.1109/I2CT.2017.8226252.
7. Z. Huiping, M. Jinbi, H. Xiaoqing, B. Liming, M. Dongjuan, and H. Jie, “A Critical Line Identification Method Based on Probabilistic Load Flow,” in 2018 IEEE International Power Electronics and Application Conference and Exposition (PEAC), Shenzhen: IEEE, Nov. 2018, pp. 1–6. Doi: 10.1109/PEAC.2018.8590644.
8. C. A. Bartend Russell and S. Khan, “Single Line Outage Analysis on IEEE 39 Bus Network,” in 2022 25th International Conference on Computer and Information Technology (ICCIT), Cox’s Bazar, Bangladesh: IEEE, Dec. 2022, pp. 821–826. doi: 10.1109/ICCIT57492.2022.10055232.
9. G. Tricarico, R. Wagle, M. Dicorato, G. Forte, F. Gonzalez-Longatt, and J. L. Rueda, “A Modified Version of the IEEE 39-bus Test System for the Day-Ahead Market,” in 2023 IEEE PES Conference on Innovative Smart Grid Technologies - Middle East (ISGT Middle East), Abu Dhabi, United Arab Emirates: IEEE, Mar. 2023, pp. 1–5. doi: 10.1109/ISGTMiddleEast56437.2023.10078548.
10. C. Chun, “A new iterative method for solving nonlinear equations,” Appl. Math. Comput., vol. 178, no. 2, pp. 415–422, Jul. 2006, doi: 10.1016/j.amc.2005.11.055.
11. O. Charity Ebelechukwu, “Comparison of Some Iterative Methods of Solving Nonlinear Equations,” Int. J. Theor. Appl. Math., vol. 4, no. 2, p. 22, 2018, doi: 10.11648/j.ijtam.20180402.11.
12. M. Čepin, “Methods for Power Flow Analysis,” in Assessment of Power System Reliability, London: Springer London, 2011, pp. 141–168. doi: 10.1007/978-0-85729-688-7_11.
13. Electric Power System Applications of Optimization, Second Edition. CRC Press, 2008. Doi: 10.1201/9781315218953.
14. William. Tinney and C. Hart, “Power Flow Solution by Newton’s Method,” IEEE Trans. Power Appar. Syst., vol. PAS-86, no. 11, pp. 1449–1460, Nov. 1967, doi: 10.1109/TPAS.1967.291823.
15. J.-H. Teng, “A modified Gauss–Seidel algorithm of three-phase power flow analysis in distribution networks,” Int. J. Electr. Power Energy Syst., vol. 24, no. 2, pp. 97–102, Feb. 2002, doi: 10.1016/S0142-0615(01)00022-9.
16. B. Stott and O. Alsac, “Fast Decoupled Load Flow,” IEEE Trans. Power Appar. Syst., vol. PAS-93, no. 3, pp. 859–869, May 1974, doi: 10.1109/TPAS.1974.293985.